An alternative concept for SMOS-HR: unfolding the brightness temperature map by along-the-track inversion of the Van Cittert-Zernike equation

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Outline

1. Introduction to the SMOS mission
2. Interferometric observations and the SMOS observation model
3. Efficient unfolding and denoising of SMOS observations with a frequency-by-frequency global inversion
4. Numerical validation of the technique
5. Conclusion

The Soil Moisture and Ocean Salinity (SMOS) satellite

- Launched in 2009. Follow-up mission SMOS-HR under consideration.
- Using passive microwave radiometry in a protected portion of the L band, it monitors two Essential Climate Variables (ECVs):
  - Soil Moisture (SM) and Sea Surface Salinity (SSS)
- In the L band, SM and SSS are responsive to observed brightness temperature, which is relatively unaffected by atmospheric conditions as L-band radiation penetrates clouds

The Soil Moisture and Ocean Salinity (SMOS) satellite

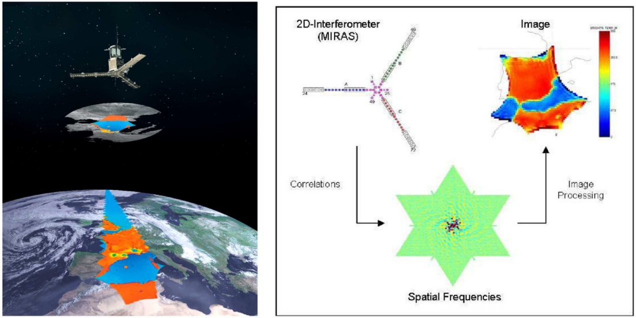
Artist rendering of SMOS.
(Image credit: CNES, ESA.)

SMOS observations

- Microwave Imaging Radiometer with Aperture Synthesis (MIRAS) instrument: three arms and 69 antennas
- Planar antenna array points toward Earth as satellite rotates thanks to a yaw correction
- Inverse problem imaging produces images of brightness temperature across directions intersecting with Earth's surface
- From these images, and other parameters (eg, sea surface temperature), SM and SSS can be recovered.

Shortcomings of current satellite

- Images produced one correlation period at a time. Images are folded (aliased) due to the undersampling of the *u*-*v* frequency plane (baselines). Images are noisy.



SMOS imaging process, with limited alias-free field of view.
(Image credit: CNES, ESA.)

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Interferometry is based on the Van Cittert-Zernike theorem

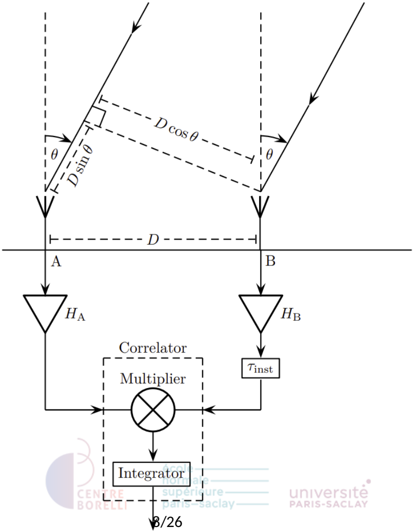
- Recall: the Einstein-Wiener-Khinchin theorem relates the *temporal coherence* (autocorrelation) of a WSS signal with its power spectrum:

$$\text{autocorrelation} \overset{\mathscr{F}}{\Longleftrightarrow} \text{PSD}$$

- The Van Cittert-Zernike theorem relates the *spatial coherence* (visibilities) associated with a far-field, quasimonochromatic, incoherent source with intensity map:

$$\text{visibilities} \overset{\mathscr{F}}{\Longleftrightarrow} \text{intensity}$$

Visibilities are sampled by correlating signals at pairs of antennas



Simplified SMOS observation model (Corbella et al.)

The visibility V_{kl} between antennas k and l may be written:

$$V_{kl} = \int_{||\xi|| \leq 1} \mathbf{T}(\boldsymbol{\xi}) h_{kl}(\boldsymbol{\xi}) e^{-2\pi i \langle \mathbf{u}_{kl}, \boldsymbol{\xi} \rangle} \, \mathrm{d}S(\boldsymbol{\xi}) \tag{1}$$

- $\boldsymbol{\xi}$ is the vector of cosine directions (ξ, η) along two orthogonal directions of the planar antenna array;
- $\mathbf{T}$ maps each direction $\boldsymbol{\xi}$ to its brightness temperature;
- $\boldsymbol{u}_{kl} = (u_{kl}, v_{kl})$ is the vector in wavelengths from antenna k to l in the u-v plane.
- $h_{kl}(\boldsymbol{\xi}) = A_k(\boldsymbol{\xi})\overline{A_l}(\boldsymbol{\xi})$, where A_k, A_l are the antenna radiation patterns of antennas k, l; and
- $\mathrm{d}S(\boldsymbol{\xi}) = \frac{\mathrm{d}\xi \mathrm{d}\eta}{\sqrt{1 - ||\xi||^2}}$ is the solid angle differential.

Simplified SMOS observation model (Corbella et al.)

This can be seen as a Fourier transform if we extend $\mathbf{T}$ to undefined directions:

$$V_{kl} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathbf{T}'(\xi) h_{kl}(\xi)}{\sqrt{1 - ||\xi||^2}} e^{-2\pi i (u_{kl}\xi + v_{kl}\eta)} \, \mathrm{d}\xi \, \mathrm{d}\eta \,,$$

where

$$\mathbf{T}'(\xi) = \begin{cases} \mathbf{T}(\xi), \text{ if } ||\xi|| \leq 1; \\ 0, \text{ otherwise.} \end{cases}$$

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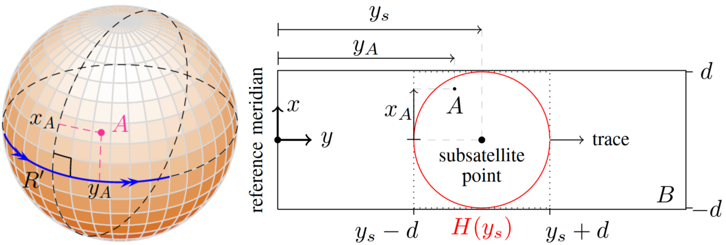
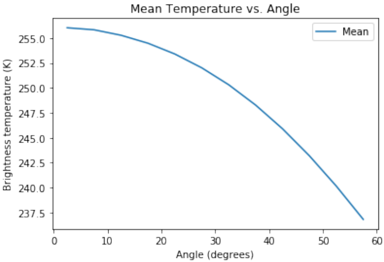
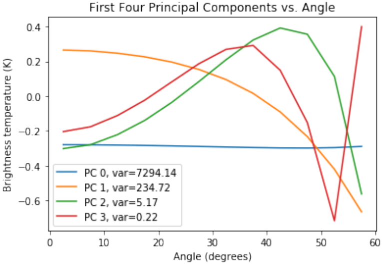
First, use geodetic coordinates *relative to the satellite's trace*, rather than direction cosines



Thanks to the satellite's yaw correction, the acquisition geometry is invariant across the orbital segment in these coordinates.

Second, characterize each (often heterogeneous!) pixel (x, y) with up to m parameters $\alpha(x, y)$





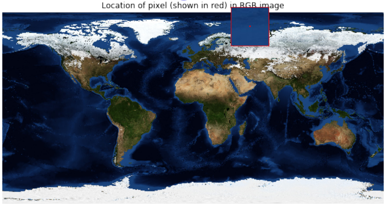
Our parametric representation of brightness temperature models:

$$\mathbf{T}(x, y, \theta) = f_0(\theta) + \alpha_1(x, y) f_1(\theta) + \ldots + \alpha_m(x, y) f_m(\theta) = \langle \alpha(x, y), f(\theta) \rangle,$$

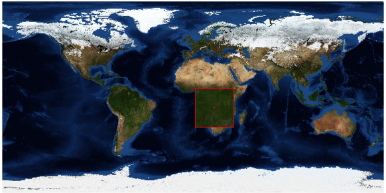
where $f = (f_i)_{i=0 \cdots m}$ are the basis functions of the model, learned from data.

Example brightness temperature curves

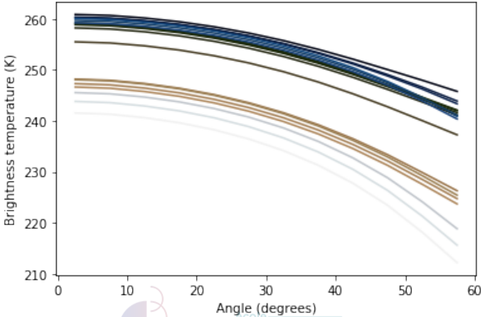
Location of pixel (shown in red) in RGB image



Location of pixel (shown in red) in RGB image



Brightness temperature curves (in their RGB color) for the eighteen sample pixels



The visibilities are now computed by convolving the image of brightness temperature parameters with a stationary function

Let

$$q(x,y) = h_{kl}(\xi(x,y))e^{-2\pi i\langle \mathbf{u}_{kl},\xi(x,y)\rangle}.(1, f_1(\theta(x,y)),\dots,f_m(\theta(x,y))).$$

The contributions to the visibilities from the visible surface of Earth can now be expressed as a convolution with respect to the subsatellite position y_s:

$$V_{kl}(y_s) = \int_{||\xi||\leq c} \langle \alpha(x(\xi), y_s + y(\xi)), q(x(\xi), y(\xi))\rangle \, \mathrm{d}S(\xi)\,.$$

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Finally, take the Fourier transform along the orbital trace

Let

$$\widetilde{V}_{kl}(\omega) = \int_{-\infty}^{\infty} V_{kl}(y) e^{-i\omega y}\, \mathrm{d}y \text{ and } \widetilde{\alpha}(x,\omega) = \int_{-\infty}^{\infty} \alpha(x,y) e^{-i\omega y}\, \mathrm{d}y$$

We see, through a simple change of variable $y' = y_s + y(\xi)$ and the stationarity of the geometry, that

$$\widetilde{V}_{kl}(\omega) = \int_{-\infty}^{\infty} \left(\iint_{||\xi||<c} \langle \alpha(x, y_s + y), q(x,y) \rangle \, \mathrm{d}s \right) e^{-i\omega y_s}\, \mathrm{d}y_s$$

$$= \iint_{||\xi||<c} \langle \widetilde{\alpha}(x(\xi),\omega) e^{-i\omega y(\xi)}, q(x(\xi), y(\xi)) \rangle \, \mathrm{d}s\,(\xi).$$

Summary

- We have written the Fourier transform of the visibilities along the orbital trace in terms of the Fourier transform of the brightness temperature parameters
- The global inversion of the observation model over all the snapshots on an orbit can now be broken down frequency by frequency. That is, for each orbital frequency ω, we can recover the transformed brightness temperature parameters $\widetilde{\alpha}(x, \omega)$
- Once we have the transformed parameters at all frequencies, we can perform an inverse transform to recover the image of brightness temperature parameters $\alpha(x, y)$
- This approach pairs the computational tractability of snapshot-by-snapshot inversions with the denoising and field-of-view widening associated with a simultaneous inversion of all snapshots

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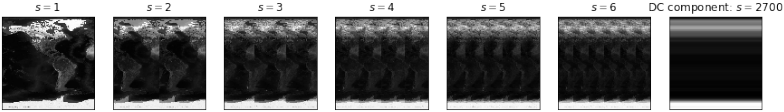
Observation: folding $\overset{\mathscr{F}}{\Longleftrightarrow}$ decimation



Lemma

Let x be an M-point discrete signal and X its M-point DFT. Suppose s divides M. Consider the $\frac{M}{s}$-point discrete signal x_f whose DFT is X, decimated by the factor s: for $\omega \in \mathbb{Z}/\frac{M}{s}\mathbb{Z}$, $X_f[\omega] = X[s\omega]$. We may write x_f as follows:

$$\text{for } m \in \mathbb{Z}/\frac{M}{s}\mathbb{Z}, x_f[m] = x[m] + x[m+s] + \ldots + x\left[m + \frac{s-1}{s}M\right]$$

Scenario: DFT sampling, folding along the trace

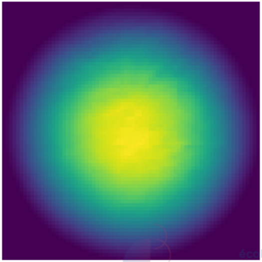
- Regularly spaced antennas along a square frame produce a regularly sampled square in the u-v plane
- We assume the modified brightness temperatures and visibilities form a DFT pair and decimate the visibilities along v-axis to introduce a highly structured pattern of aliasing that can be directly modeled and unfolded
- Individual snapshots cannot be unfolded (there is no alias-free field of view) but a set of snapshots along an orbital segment can be unfolded two ways:
  - Using a direct model of the folding mechanism (concatenated circulant matrices operating on the brightness temperature parameters)
  - Using the partial Fourier transform method

Sample inversion

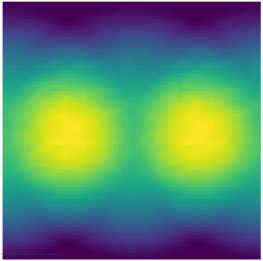
Image of Brightness Temperature (BT) Parameters

Recovered BT Parameters ($s = 2$, $C = 3$, $B \cdot \tau = 2 \times 10^7$)

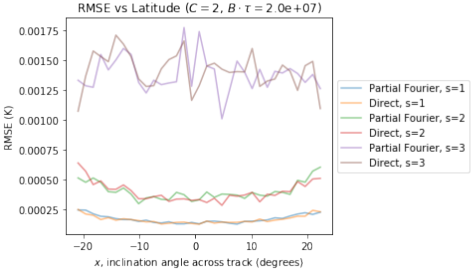
BT Image of Central Snapshot ($s = 1$)

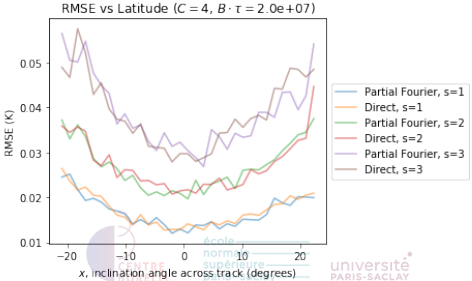


BT Image of Central Snapshot ($s = 2$)



Method comparison





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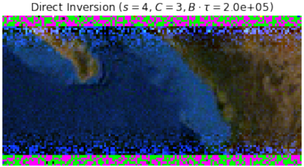

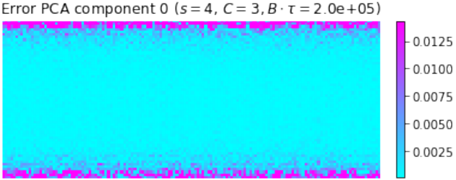

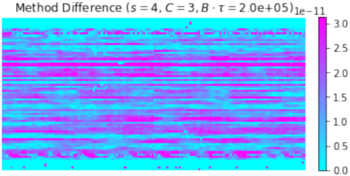

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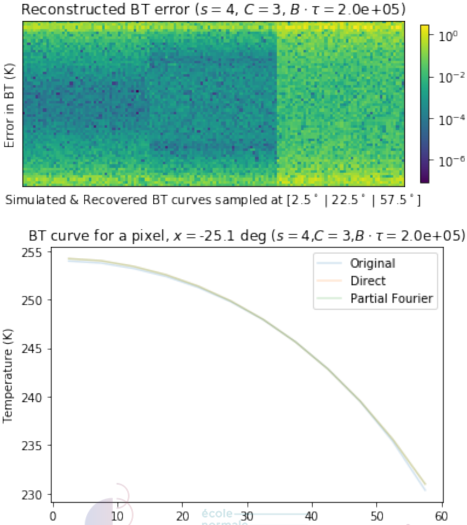

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Conclusion

- We introduce a new approach to inverting the SMOS observation model that marries the denoising and field-of-view widening of a global inversion with the computational tractability of a snapshot-by-snapshot inversion
- This approach is enabled by the invariance of the acquisition geometry in geodesic coordinates across the orbit
- This inversion technique enables new array designs that use repeated observations of Earth to compensate for aliasing (undersampled baselines) in the direction of the orbit