Physicists on Tap

Surprising Applications of Optics

Keystone Oaks High School

Max Dunitz (he/they), PhD Student at Centre Borelli, France

March 8, 2023

Table of Contents

1 Sensing EM Radiation

I Sensing EM Radiation

I EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

I Mapping Sound Waves to the Airwaves with the Cosine Product Rule

I Is the Moon Made of Green Cheese?

I Soil Moisture and Ocean Salinity Satellite

I Big Polluters Don't Want You to Know about This One Weird Trick: Multispectral Methane Detection

I The EM Spectrum: A Finite Public Good

I Unsolicited Career Advice

Sensing Visible Light

- Like emulsion layers in film[1] or sensors on a digital camera, your eyes have three different color sensors.

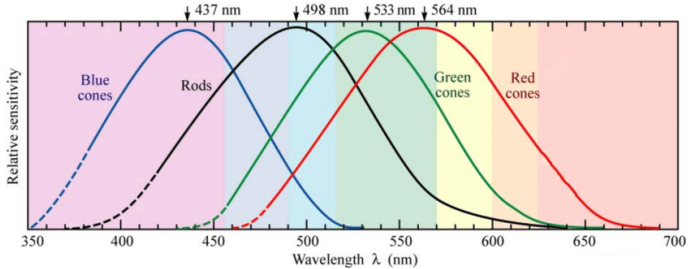


Figure: Source: Dowling, John E. *The retina: an approachable part of the brain.* Harvard University Press, 1987.

[1] Color film has blue, green-and-blue, and red-and-blue layers. Certain films (like the original Ektachrome) have two layers.

Digital Cameras

1 Sensing EM Radiation

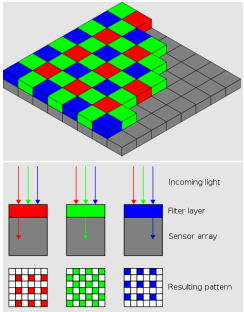


Figure: Bayer pattern in a digital camera sensor. Our eyes are more responsive to green than red or especially blue so the camera captures more information in this channel. Source: Jonathan Thomson's weblog.

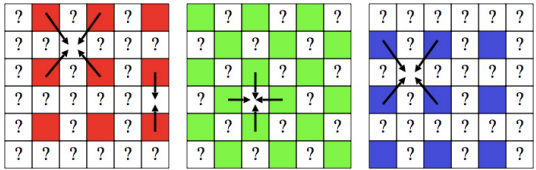


Figure: In a process known as *demosaicking*, missing values are imputed via a weighted average of the neighboring measured values in the same color channel. Every pixel thereby obtains a red, green, and blue value, even though its associated sensor captures only one color. Source: Svetlana Lazebnik, UIUC.

Film Cameras

1 Sensing EM Radiation

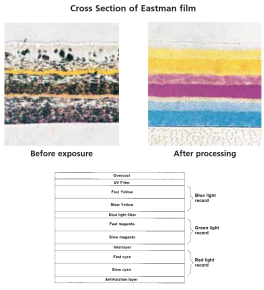


Figure: Emulsion layers in film. The blue layer is exposed first because all forms of silver halide are sensitive to blue light. Source: Exploring the Color Image (Kodak).

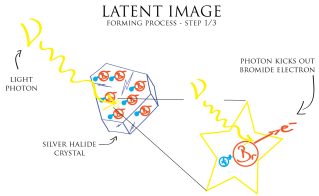


Figure: Photons free electrons from Br in the AgBr crystal, which then accumulate in crystal imperfections, attracting Ag^{+}. Developer then accentuates these differences by turning this silver seed into metallic silver. (An ultraviolet filter is used in film photography because silver halide is sensitive to ultraviolet light too.) Source: Film Photography weblog.

Phototransduction

1 Sensing EM Radiation

- When photons of the correct wavelength strike 11-*cis*-retinal (similar to vitamin A) in opsin proteins, it straightens out into an all-trans configuration. This configuration change triggers a chemical signaling cascade.

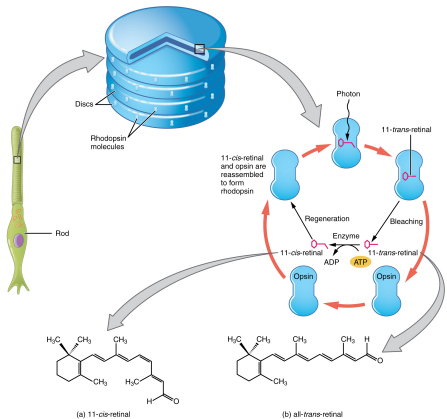


Figure: Source: Anatomy & Physiology, Connexions Web site.

What Happens When You Look up at a Cloud?

- 10^{16} photons pierce each eye each second. Since the clouds reflect red, green, and blue light in roughly equal measure, thse photons are of wavelengths associated with red, green, and blue visible light in roughly equal measure.
- 10^{15} photons reach each retina, lined with vision cells.

Figure: Agnès Varda, still from *The Beaches of Agnès*.

Dynamic Range of Sight

- In bright light, the cone cells function well; in dim light, they are too unreliable (they have a high *signal-to-noise ratio*) and we depend only on rods.

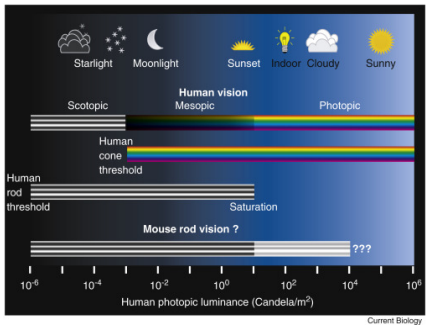


Figure: The *dynamic range* of the eye. It is quite remarkable that the eye can respond to dim starlight and yet not be overwhelmed by the cacophony of light on a bright day. Source: Kelber, Almut. "Vision: Rods see in bright light." *Current Biology* 28.8 (2018): R364-R366.

Dynamic Range of Sight

- Frog eyes can see a single photon.[2]As you move a point light source farther and farther away from the frog, it does not become invisible but is seen less infrequently as photons arrive less frequently.

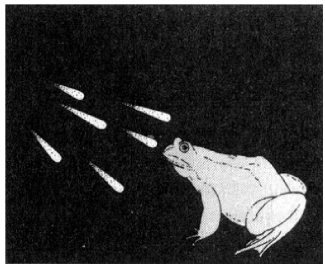


Figure: Source: David Deutsch, *The Fabric of Reality*.

[2]Certainly their rods respond in laboratory conditions to single photons. As cold-blooded animals, their vision is less susceptible to thermal noise. Humans require more photons to elicit conscious response.

What Happens When You Look up at a Cloud?

- Only a small fraction of photons strike chemically available photopsin proteins. Those that do trigger the phototransduction cascade:
  - Gated sodium channels spend more time closed; with less sodium entering, the cone cells hyperpolarize, generating fewer action potentials, and thus more action potentials downstream, in the retinal ganglion cells, whose axons form the optic nerve and carry the signal to the back of the brain.
- This chemical filtering condenses the image to a metabolically manageable 10^7 bits/second. Still, the brain relies on many tricks to process these data.

Since the clouds reflect red, green, and blue photons in roughly equal measure, the cone cells respond in roughly equal measure, and typically functioning optical systems create a perception of the cloud as “white.”

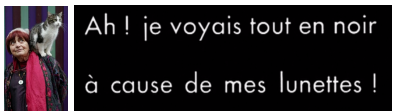


Figure: Source: Raindance, intertitle from the movie-in-a-movie in *Cleo from 5 to 7*.

A Clarification: “Thermal Radiation”

- All EM radiation heats, not just infrared radiation. Unlike conduction and convection, radiation can travel through the vacuum of space, which is how our planet is warmed (and emits heat).
- EM radiation of any frequency increases with the source’s temperature. This is why EM radiation is called “thermal radiation.”
  - The relative proportions change, however; this is the difference between “red hot” and “white hot.”

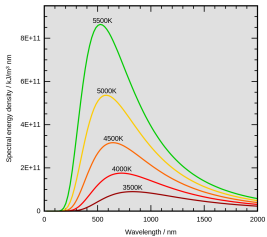


Figure: Source: Wikipedia user 4C.

A Clarification: “Thermal Radiation”

- Most of the radiation our bodies emit is infrared (12 micron wavelength, just longer than visible red).
  - Hence the use of *infrared cameras*: they enable the surveillance of people at night.
- We *absorb* infrared well too. (Kirchoff’s Law: bodies tend to emit and absorb at the same frequencies.)
  - Wood-burning fires are a source of infrared radiation. The warmth of a fire is largely from near-infrared radiation heating our body. This radiation does not penetrate the body of someone standing between you and the fire.

Figure: Iron, at high temperatures, emits a lot of visible light. (It also emits infrared; be careful!) Most visible light we see (apart from screen time) is reflected; we tend not to look at major visible light sources due to their intensity. Source: Wikipedia user Fir0002.

Table of Contents

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

I Sensing EM RadiaŒon

I EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

I Mapping Sound Waves to the Airwaves with the Cosine Product Rule

I Is the Moon Made of Green Cheese?

I Soil Moisture and Ocean Salinity Satellite

I Big Polluters Don't Want You to Know about This One Weird Trick: MulŒspectral Methane DetecŒon

I The EM Spectrum: A Finite Public Good

I Unsolicited Career Advice

Glass Is Opaque to Infrared RadiaŒon

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

Figure: SŒll from video[4] Hot co€ee mug seen through glass in the visible spectrum (photo camera, le‰) and infrared (infrared camera, right). NoŒce how the glass is transparent to visible light and opaque to infrared radiaŒon.

[4] Source: YouTube.

A Greenhouse

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

Figure: Source: UK Dept for Environment, Food, & Rural A€airs.

Figure: Visible light passes through greenhouse gases in the atmosphere, warms the surface of Earth, which radiates mostly infrared radiaŒon. But infrared radiaŒon interacts with greenhouse gases in the atmosphere: the heat is \trapped."

Atmosphere Pro•le Due to Thermal RadiaŒon

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

Figure: With the excepŒon of the thermosphere (which is hot due to convecŒve heat transfer between atmospherical parŒcles and solar wind), the temperature pro•le of the atmosphere can be predicted from radiaŒon absorpŒon spectra. Source: Kerry Emanuel.

- Diatomic O_2 and N_2 are poor absorbers or radiaŒon. If the atmosphere were composed of only these two gases, the Earth would be about 33 degrees Celsius colder. The major greenhouse gases are triatomic (water vapor, ozone, CO_2), though trace gases like methane (CH_4) can be highly potent greenhouse gases.

Solar RadiaŒon Peaks in Visible Spectrum, Earth in Infrared

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

Figure: Source: Ahrens, C. D. (200Ž). EssenŒals of meteorology: an invitaŒon to the atmosphere. Wadsworth.

- Earth and the Sun are at very di€erent physical temperatures. They emit/absorb with peaks at di€erent wavelengths. The Sun emits at much greater intensity than does Earth.

AbsorpŒon of RadiaŒon from the Sun

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

- Most of the high-energy, DNA-destroying ultraviolet radiaŒon is absorbed in the stratosphere through a photochemical reacŒon in which oxygen splits and forms ozone. The ozone also absorbs ultraviolet radiaŒon.
- The high temperatures of the upper stratosphere are due to the absoprŒon of UV radiaŒon by ozone.
- Emission of hydro,uorocarbons diminished this ozone layer, but collecŒve global acŒon through the Montreal Protocol has allowed the layer to start recovering.

Figure: The ozone layer is quickly recovering and if progress conŒnues should be fully recovered b
2040-2070.

Albedo

- A simple model of Earth says that it re,ects a proporŒon of 30% of the radiaŒon from the sun that reaches the upper atmosphere and then absorbs and radiates the rest like a black body.
- This fracŒon is called the *albedo* and depends on weather condiŒons, and ground condiŒons.
  - The peak of solar radiaŒon is visible spectrum. White surfaces re,ect all visible light. Thus, clouds and snow increase the albedo of Earth.

Figure: White buildings in Greece keep cooler in the summer sun. Some scienŒsts have proposed painŒng structures white systemaŒcally to boost Earth's albedo. Source: Santorini View.

Stefan-Boltzmann Law

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

- Earth absorbs all non-re,ected radiaŒon from the Sun (i.e., it is opaque). Kircho€'s law: Earth emits the radiaŒon it absorbs.
- Using measurements of the intensity of solar radiaŒon as well as the fact that Earth emits radiaŒon in all direcŒons even though it is illuminated from one, we can esŒmate the radiaŒon Earth emits has intensity $I = 238{:}9\ \mathrm{W\,m^{-2}}$.
- Stefan-Boltzmann law: the intensity of radiaŒon emi‹ed by a black body is proporŒonal to the fourth power of its temperature: $I = T^4$, where the proporŒonality constant $= \frac{2\ ^5k^4}{15c^2h^3} \quad 5{:}67 \quad 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$ depends on Planck's constant h, Boltzmann's constant k, and the speed of light c.
- Solving for temperature, we get

$$T = \frac{238.9\ \mathrm{Wm^{-2}}^{\frac{1}{4}}}{5{:}67 \quad 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}^{\frac{1}{4}}} \quad 255\ \mathrm{K}$$

The Greenhouse E€ect

2 EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

- Earth's actual surface temperature is 288 K|and rising.
- This extra 33 degrees Celsius (288 K - 255 K) is due to the greenhouse e€ect.
  - Earth is not a perfect black body. It does not emitsotropically, or the same at all direcŒons. (Thanks to this non-uniformity, satellites can see structure in the Earth in many bands!) However, this violaŒon of the hypothesis of the Stefan-Boltzmann law turns out to not really a€ect the math.

- Even if the Sun, the Earth, and the layers of the atmosphere are in radiaŒve equilibrium, they are not at the same physical temperatures. In parŒcular, the surface air is not in thermal equilibrium with the Earth's surface. This enables conducŒve and convecŒve heat transport|the basis of weather in the troposphere.

Table of Contents

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

I Sensing EM RadiaŒon

I EM Waves in Media: Why Glass and the Atmosphere Are Not Always Clear

I Mapping Sound Waves to the Airwaves with the Cosine Product Rule

I Is the Moon Made of Green Cheese?

I Soil Moisture and Ocean Salinity Satellite

I Big Polluters Don't Want You to Know about This One Weird Trick: MulŒspectral Methane DetecŒon

I The EM Spectrum: A Finite Public Good

I Unsolicited Career Advice

Remember this? I can't.

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

The Cosine Product Rule

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x \quad y) + \cos(x + \ y))$$

We prove it using Euler's theorem.

Remember this? I can't.

The Cosine Product Rule

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x \quad y) + \cos(x + \ y))$$

We prove it using Euler's theorem.

Euler's Theorem

$$e^{ix} = \cos(x) + \ i\sin(x)$$

Remember this? I can't.

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

The Cosine Product Rule

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x \quad y) + \cos(x + \ y))$$

We prove it using Euler's theorem.

Euler's Theorem

$$e^{jx} = \cos(x) + \ i\sin(x)$$

First, we derive a fancy way to write $\cos(x)$: Consider the expression $e^{jx} + e^{j(\ \ x)}$:

$$e^{jx} + e^{j(\ \ x)} = (\cos(x) + \ i\sin(x)) + (\cos(\ \ x) + \ i\sin(\ \ x))$$

$$= (\underbrace{\cos(x) + \cos(\ \ x)}_{2\cos(x)\text{ as cos is even}}) + i(\underbrace{\sin(x) + \sin(\ \ x)}_{0\text{ since sin is odd}}) = 2\cos(x):$$

Divide both sides by 2 and we get…

Remember this? I can't.

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

A Fancy Way to Write $\cos(x)$

$$\cos(x) = \frac{e^{ix} + e^{i(-x)}}{2}:$$

Now take the product $\cos(x)\cos(y)$ a‰er pu“ng them in this fancy form:

$$\cos(x)\cos(y) = \frac{e^{ix} + e^{i(-x)}}{2}\,\frac{e^{iy} + e^{i(-y)}}{2} = \frac{1}{4}\left(e^{ix} + e^{i(-x)}\right)\left(e^{iy} + e^{i(-y)}\right)$$

$$= \frac{1}{4}\left(e^{ix}e^{iy} + e^{ix}e^{i(-y)} + e^{i(-x)}e^{iy} + e^{i(-x)}e^{i(-y)}\right)$$

$$= \frac{1}{2}\left(\frac{e^{i(x+y)} + e^{i(-(x+y))}}{2} + \frac{e^{i(x-y)} + e^{i(-(x-y))}}{2}\right)!$$

$$= \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$

The Frequency Domain

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

- A signal can be decomposed into a sum of sinusoids of different frequencies, the way a symphony can be broken down at any moment in the different notes being played.
- We can write signals as sums of sinusoids:

$$\sum_{i=1}^{N} \underbrace{a_k \cos(2 f_k t)}_{\text{even part}} + \underbrace{b_k \sin(2 f_k t)}_{\text{odd part}}$$

- The Fourier transform expresses a signal in terms of the weights $(a_k; b_k)$ on these sinusoids: as a single complex number $w_k = a_k + ib_k$

Figure: Source: Stack Exchange.

Fourier Synthesis

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

- Loosely speaking, the more frequencies present in the signal, the more expressive it can be. This is why we use *bandwidth* to refer to information content: more frequencies are required.

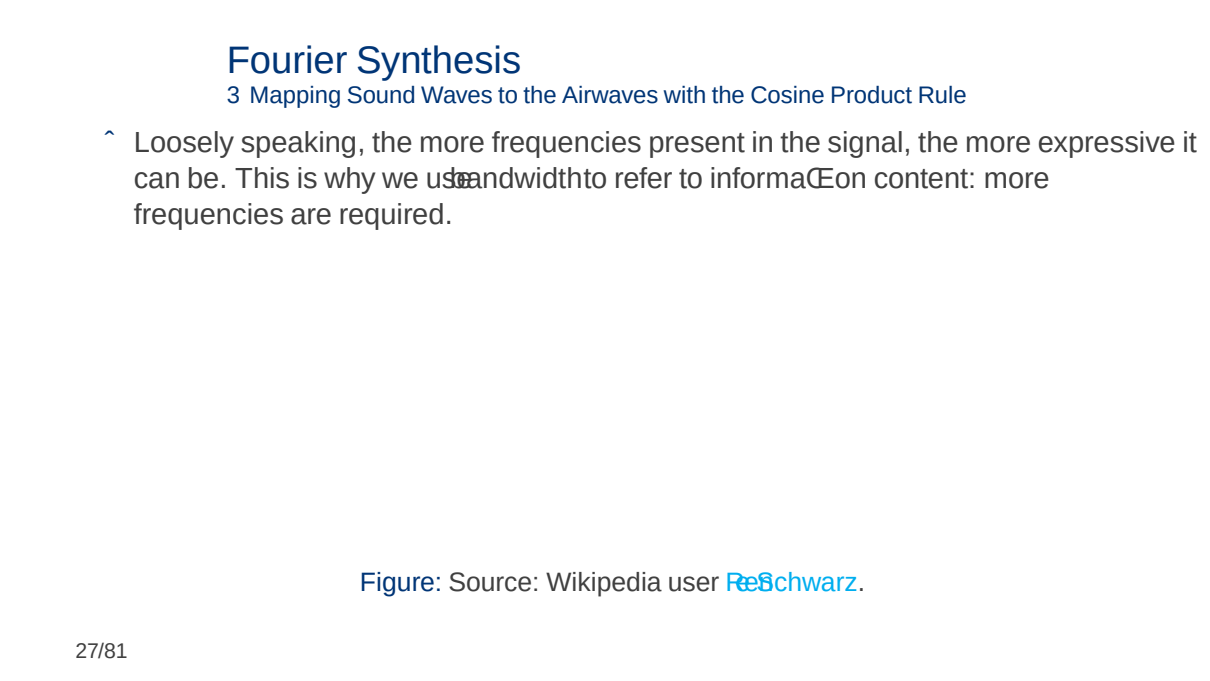


Figure: Source: Wikipedia user Ree Schwarz.

Engineering: Pu“ng Math to Use

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

Suppose our sound signal S is a pure tone of f_s cycles per second:

$$S(t) = \cos(2\ f_s t): \text{ (If } S \text{ is audible, } 20 \text{ Hz} \quad f_s \quad 20000 \text{ Hz.)}$$

Suppose our carrier tone C is f_c cycles per second (so that it travels well through the air):

$$C(t) = \cos(2\ f_c t): \text{ (} C \text{ is adapted to AM radio communicaŒons channel, 500-1600 kHz}^5\text{)}$$

Our transmi‹er mulŒplies the two tones for transmission over the airwaves

$$T(t) = S(t)\ \ C(t) = \cos(2\ f_s t)\ \ \cos(2\ f_c t) = \frac{1}{2}(\cos(2\ \ (f_s + f_c)t) + \cos(2\ \ (f_s \quad f_c)t))$$

[5] Typically marketed as the kHz frequency. For example, KDKA 1020 newsradio is at 1020 kHz. AM radio carries less informaŒon than FM and is typically limited to voice applicaŒons (news, traf c, call-in, sports, religion, etc.) with only \simple" pop music. FM is more adapted to jazz, classical, more complex pop music, avant-garde traf c: it has more bandwidth.

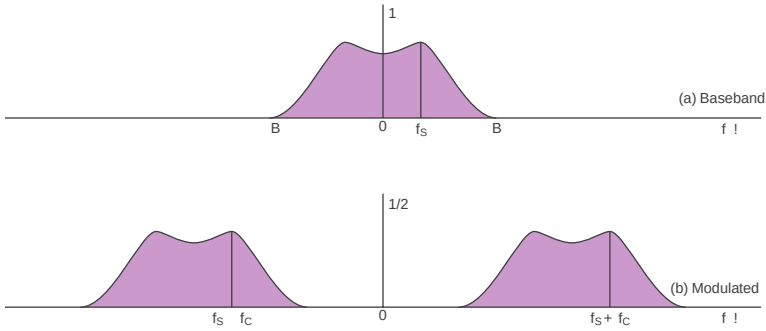
Engineering: Pu“ng Math to Use

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

We mulŒplied our sound signal $S(t)$ (a sinusoid of frequency f_S) by the carrier tone $T(t)$ (a sinusoid of frequency f_C) and now we have two scaled sinusoids: one of frequency $f_S + f_C$ and one of frequency $f_S - f_C$ (which is negaŒve).

QuesŒon

We want to get our signal S back. What happens when we mulŒply by T again?



We get our original signal back (scaled by 1/2), plus some \junk":

$$T(t) \quad (S(t)T(t)) = \cos(2 \quad f_c t) \quad \frac{1}{2} \quad \cos(2 \quad (f_s + f_c)t) + \cos(2 \quad (f_s \quad f_c)t)$$

$$= \frac{1}{4} \quad \cos(2 \quad (f_s + 2f_c)t) + \cos(2 \quad (f_s + f_c \quad f_c)t) +$$

$$\cos(2 \quad (f_s \quad f_c + f_c)t) + \cos(2 \quad (f_s \quad 2f_c)t)$$

$$= \underbrace{\frac{1}{4}(\cos(2 \quad (f_s + 2f_c)t) + \cos(2 \quad (f_s \quad 2f_c)t))}_{\text{high-frequency components to •lter out}} + \underbrace{\frac{1}{2}\cos(2 \quad f_s t)}_{\frac{1}{2}S(t)}$$

AM Transmi‹er

3 Mapping Sound Waves to the Airwaves with the Cosine Product Rule

Figure: Amplitude ModulaŒon: mulŒplicaŒon by a carrier tone. Source: Rod Nave.