The Novel Electoral System Introduced in the 2020 US Democratic Party's Presidential Nomination Process Subverts Proportional Representation*

Max Dunitz
Université Paris Saclay, Université Paris Cité, ENS Paris Saclay, CNRS, SSA, INSERM, Centre Borelli, F-91190, Gif-sur-Yvette, France, max.dunitz@ens-paris-saclay.fr

In 2020, the US Democratic Party began experimenting with a novel electoral system, based off that of the party-run caucuses. In each contest, this electoral system introduces instant-runoff voting before the proportional representation apportionment of the contest's delegates using Hamilton's method. Four state parties (those of Alaska, Hawaii, Kansas, and Wyoming) used this system in their contests (party-run primaries in the case of first three states, and a party-run caucus in the latter); all appeared late in the nomination process, after Joe Biden had become the presumptive nominee. Ranked-choice voting was also used in absentee ballots for the Nevada caucus. Proponents of this system argue that it can reduce the phenomenon of *wasted votes* that are particularly prevalent due to the Democratic Party's use of 15% thresholds for delegate accumulation (regardless of the size of the contest's delegate pool) and the extension of early and absentee voting. In this paper, we examine this novel electoral system in greater detail and consider how ranked-choice voting can interact with proportional representation to create *strongly* non-leader outcomes, whereby a candidate A is preferred by a majority of voters to B, and yet B can be apportioned *many* more delegates than candidate A. We also generalize the no-show paradox to delegate allocations by defining a new partial order of allocations in terms of up-preference delegate flows and observe that this novel system is highly susceptible to the no-show paradox. Finally, we examine how this new source of non-leaderness interacts with existing non-leaderness due to district-level delegate allocation.

Key words: elections, voting, Democratic Party, leader paradox, non-monotonicity, no-show paradox, ranked-choice voting, aggregation paradox, RCV, IRV, proportional representation, majorization, fairness, delegate transfers

History: This paper was based on a blog post on the author's website written April 20, 2023, and updated in May. It was submitted to SSRN on September 4, 2023.

1. Introduction

The 57 contests for the Democratic presidential nomination in the US use a modified form of proportional representation (PR). Whether these contests are party-run primaries or caucuses, or a

* Disclosure: the author helped administer the 2020 Democrats Abroad primary (including tabulation of both remote and voting center balots) and is a voting member of the Democratic Party Committee Abroad (DPCA) who on April 22, 2023, voted (reluctantly) for a DPCA Delegate-Selection Plan that includes instant-runoff voting. The former chair of the DPCA who secured this change now works at FairVote.

party-run PR tabulation overlay on a state-run first-past-the-post-style primary election,[1] the apportionment of delegates is performed by the state party after its state contest using PR, with a 15% quota and Hamilton's largest-remainder method used to assign any remaining delegates, in accordance with their Delegate-Selection Plan (DSP). The DSP may allow delegates to be performed in district-level subcontests in addition to a state-level contest, a process that can produce paradoxical outcomes (Jones et al. 2019).

This 15% threshold is rather high by international standards (Gallagher 1992), and, as proponents of ranked-choice voting system argue (Richie et al. 2021), can lead to tactical voting and voter remorse. With the expansion of early and absentee voting, early voters in particular can have difficulty judging how to vote tactically in a fast-moving race.[2] The DSPs of the state Democratic Party chapters in Alaska, Hawaii, Kansas, and Wyoming included instant-runoff voting (IRV) before proportional representation tabulation as a kludge to mitigate voter discontent with this high threshold, since altering the 15% threshold is difficult under party rules. Several state parties, including the Democratic Party Committee Abroad and the North Dakota Democratic-Nonpartisan League Party, are slated to join them in 2024 (DPCA 2023, ND Dem NPL 2023). The ranked-choice ballots for these party-administered elections allow voters to rank multiple candidates. Instant-runoff voting is used to ensure that each ballot expressing complete preferences will be added to a candidate's total before proportional representation apportionment, that is, no ballot expressing complete preferences will contribute to the final-alignment vote total of a candidate falling below the threshold of 15% used in the proportional representation allocation. (If no candidate receives 15% of the votes cast in a particular state- or district-level contest, the threshold is set at half the vote total of the front-runner.) Using instant-runoff voting, candidates with the fewest votes are successively eliminated and their votes redistributed to the next preference on the ballot until all remaining candidates attain the threshold, whereupon the delegate haul is apportioned among them using Hamilton's largest-remainder method.

In this article, we argue that this novel combination of ranked-choice voting and proportional representation has a tendency to produce paradoxical outcomes because the patch for the high threshold fails to account for the interactions between the two electoral systems cohabiting in the DSPs. In the remainder of this section, we examine the effect of an aggregation paradox in the 2020 Iowa Democratic caucuses and discuss the introduction of the novel electoral system in 2020. In section 2, we demonstrate the susceptibility of the novel system to non-monotonic outcomes like

[1] As more states adopt ranked-choice voting systems, state-run primaries may develop the capacity to produce and tabulate ranked-choice ballots for presidential primaries.

[2] Among state elections—overseen by paid clerks with permanent offices that are regularly made accessible to voters—currently only Michigan allows voters to "spoil" a regretted early vote and revote. For party-run contests, the administrative burden imposed by revotes is seen by volunteers as prohibitive.

the no-show paradox using detailed tabulation examples. In section 3, we generalize two definitions used in the literature to compare the performance of proportional representation with the the novel system introduced in the 2020 primaries that combines proportional representation with instant-runoff voting. Finally, in section 4, we make our simulations more realistic by adding district-level races, to see how the paradoxes of the novel system interact with aggregation paradoxes.

1.1. The Real 2020 Iowa Debacle: Failure to Adhere to the Leader Criterion

Certain questions in US politics provoke the Great Doubt, like *Who won the 2020 Iowa Democratic Caucuses?*

In a protracted nomination process, there is no nominee on an early primary or caucus night. Each campaign's aim is to be declared the winner of the individual state's contest by most news media outlets; the delegates accumulated are on the own incidental, at least in the early contests leading up to Super Tuesday. The earned media and exhibition to voters and donors of "electability"—or, at least, organizing and campaigning capacity—are the real prizes. In 2020, to some extent, Pete Buttigieg won these, but he did not get the most votes.

In 2020, the Iowa Democratic Party reported two results: the raw counts of first-alignment and final-alignment raw votes, as well as the tabulated State Delegate Equivalents (SDEs). It is this latter figure that most major news outlets used to determine the winner, for in past contests, this was the sole figure reported. While Sanders won the first-alignment and final-alignment raw votes, the certified results accord Pete Buttigieg more SDEs by an inappreciable margin, and thus Buttigieg "won the caucus."[3]

It takes work to make a proportional representation violate *the leader criterion*, that is, the principle that the candidate with the most votes should get the most delegates. In general, we will call a multi-candidate race non-leader if some candidate A gets more votes than candidate B, but B receives strictly more delegates than A. The Iowa Democratic Party achieves this feat of unfairness with cascading rounding error and—more significantly—geographic weighting of votes.[4] Well-meaning

[3] In the 2020 Iowa caucus, the Associated Press made the rare decision to not declare a winner (of SDEs) because, even after a limited recount, the Iowa Democratic Party did not fix known tabulation errors, missing precinct data, and inconsistencies (Belin 2020) and certified incorrect results (Associated Press 2020). Buttigieg was assigned 4.3×10^{-2} percentage points more SDEs (i.e., the margin in SDEs as a percentage of the total number of SDEs awarded), which the AP deemed too close to call given the errors and inconsistencies. Nevertheless, headlines that Pete Buttigieg "won the caucus" were widespread in the news media, especially after the Iowa Democratic Party released nearly complete results (see, e.g., Becker and Martina (2020)).

[4] The state convention awards the 41-member delegation to the DNC, which comprises state delegates (which Buttigieg ultimately won by 1) and district delegates (which Buttigieg also won by 1). These are apportioned to candidates based on their SDEs, which depend on the location of the caucus site. Counties get a fixed number of delegates to the state convention to elect the DNC delegates, based on the number of votes the last Democratic gubernatorial and presidential candidates got in that county. Delegates to the state convention are chosen at county conventions. Counties set the size of their own county conventions, and they aren't weighted by the county's population or number of Democratic voters in the county; they can be set by the capacity of the meeting location. SDEs are computed at

attempts to promote geographic diversity or ensure more people who want to participate in exciting party conventions can get a chance to do so[5] ultimate produce an illegible outcome and a system that is difficult to administer. In a statement for the Sanders campaign, Jeff Weaver called the SDEs "an antiquated and meaningless metric" (Fedor 2020). Sanders-friendly opinion columnists pointed out the inconsistencies between Buttigieg's statements in favor of the National Popular Vote Interstate Compact and basic democratic priniciples and his claiming of victory using weighted votes (Day 2020, Dickinson 2020).

The resulting paradoxical outcome was seen by many—with good reason!—as a violation of democratic standards and, in particular, the leader criterion. Comparisons to, notably, the Electoral College's non-leader aggregation in 2000 and 2016 propagated on social media. The hashtag #MayorCheat trended on Twitter, as accounts purporting to be upset Sanders supporters or Republicans promoting disaffection attached it to tweets like, "Are my eyes deceiving me or does Bernie not have more votes then #MayorCheat yet they have Mayor Pete in first place. Only in democrat math can 2nd be 1st 🤣😂🤣"(Real Talk News1 2020). Accounts posting anti-Sanders content retorted that, in 2016, Sanders's anti-superdelegate stance and success in low-turnout caucuses betrayed previous support for an outcome that would not adhere to the leader criterion (Real KHive Queen B 2020). At the same time, the spectacular failure of the results-reporting application IowaReporterApp[6] delayed the results; giving further fodder for conspiracy theories, the CEO of Acronymn, the parent company of Shadow Inc., the vendor contracted to produce IowaReporterApp, was married to a Buttigieg staffer (Biesecker and Slodysko 2020). On social media, accusations of a "rigged" primary swirled, with Donald Trump's children tweeting, "The fix is in... AGAIN" (Donald Trump Jr. 2020) and

caucus sites based on the pre-determined number of county convention delegates assigned to the caucus site, and the pre-determined number of state convention delegates assigned to the county convention. All this means that not all votes are weighted equally. If Sanders can successfully appeal to voters in areas that don't usually turn out or don't usually vote Democratic, his supporters' votes will count less. In fact, this paradoxical outcome likely happened in 2016 as well: while Bernie Sanders did win the one (low-turnout!) county (Johnson, home of the University of Iowa) that Jack Hatch won in 2014, he did win the majority of Terry Branstad's best counties, as well as those where turnout was lowest (The Iowa Legislature 2016). It is for this reason that Bernie Sanders's campaign requested, via the press, the raw vote counts from the party (Roberts 2016).

[5] Party rules are written with input from party volunteers, who like attending meetings, and not normal people, who like understanding who won the election! Delegation sizes are selected before the caucus, rather than after the caucus, when more fair delegation sizes could be computed. In effect, much of the focus of a DSP is situated on the selection of delegates fairly from the pool of party volunteers (and gadflies), rather than the caucus or primary winner fairly from the pool of votes. (Disappointed volunteers complain to your face, whereas disappointed voters complain on social media.) Under these provisions, generally low-Democratic-turnout regions are assumed to have relatively inactive party chapters and fewer people who want to attend the conventions, whereas reliably high-Democratic-turnout regions are assumed to have active local party chapters, with a larger pool of dedicated Democratic voters to recruit from (in part due to chapter work). They get a delegate boost above the presidential primary turnout because larger delegations are needed to accommodate the huge demand for attending meetings.

[6] While it was a closed-source app distributed fairly widely with multiple security flaws and clearly produced by inexperienced labor from a contractor Shadow Inc. (rather than the now largely unionized party staff workers), perhaps its biggest flaw was that it was deployed late, with insufficient time to test and train volunteers to use its complicated interface (Koebler et al. 2020).

"Mark my words, they are rigging this thing ... what a mess" (Eric Trump 2020), and his then-campaign manager tweeting, "Quality control = rigged? 🤔" (Parscale 2020). While there was no doubt a great itch in the Trump camp to disseminate baseless conspiracies and amplify divisions in the party, this non-leader-satisfying outcome—as well as technically leader-satisfying but unfair outcomes, such as the apportionment of delegates in Des Moines's precinct 80, where a math error transformed a 101-66 Sanders-Buttigieg final-alignment vote split into a 4-4, rather than 5-3 (per the DSP), Polk County convention delegate split (Linkletter 2020)—animated much of the clamorous reaction to the caucus tabulation results.

The tabulation complexity, need for party volunteers to turn to an unproven vendor to develop an app, and delay in results all stem from the party's choice to use an unnecessarily complicated tabulation system—one that breaks monotonicity. Party volunteers adopted complicated rules that produced perverse outcomes and undermined confidence in their volunteer-administered, party-run elections. The widespread outcry and spread of conspiracy theories have their roots in the DSP.

1.2. Proportional Representation and Ranked-Choice Voting in the Democratic Presidential Primaries

Since the abolition of the "unit rule" (whereby state delegations voted as a unit for the candidate supported by the majority) at the 1968 Democratic National Convention, the Democratic Party presidential nomination process has used some form of proportional representation (Sánchez 2020). The quota for obtaining delegates in contests was lowered to 15% in all states in 1992, after the contentious 1980 primary campaign and subsequent Hunt Commission report had opened the conversation a decade earlier (Aldrich 2009). It has remained at 15% since.

Per the Delegate Selection Rules, states have wide latitude to modify their DSPs to include what amount to geographic weighting schemes[7] or instant-runoff tabulation, but "no state shall have a threshold above or below fifteen percent" (Democratic National Committee (2022), Rule 14), except if no candidate meets that threshold in the state- or district-level contest. These alterations can substantially affect the mathematical properties of the electoral system—in some situations, more than the threshold itself.

Of course, geographic weighting already exists in the drawn-out nomination process: each state holds a separate contest, awarding a number of delegates that is determined not after the last vote in the last state is counted, but in fact before voting begins. Thus, susceptibility to aggregation paradoxes is embedded in the nomination process. It is the price paid to incentivize retail politics

[7] These weighting systems are sincerely meant to satisfy the demands of some in the party to participate in more meetings, but cascading rounding error and the selection of district-level delegation sizes before the election takes place (per Democratic National Committee (2022), Rule 8) induces geographic weighting and potentially leads to elimination and aggregation paradoxes (Jones et al. 2019).

over television ads and to put state campaign operations to the test throughout a grueling primary calendar ahead of a general election that will be decided in just a handful of states—without ceding all evaluation of progress toward securing the nomination to the turnout projection models of data journalists. But by leaving state parties free to choose from a variety of electoral systems that preserve the 15% threshold for proportional representation, the Democratic Party introduces vulnerability to intra-state weighting- or tabulation-related paradoxical outcomes by, for instance, holding district-level contests based on vote tallies in each district in prior elections or by inserting IRV tabulation in the proportional representation system. These illegible and paradoxical outcomes impair voters' confidence in the process.

Recent reform movements have focused on reducing the influence of unpledged delegates on the nomination process and minimizing the role that caucuses, with their high barriers to participation, play (Jewitt 2019). Over time, caucuses have become more accessible due to standardization[8] and the introduction of satellite caucuses and even absentee ballots with ranked-choice voting, as Nevada did in 2020. The natural way to make caucuses entirely accessible is to replace them with party-run primaries, while preserving some form of realignment with instant-runoff voting tabulation—at the state or district level, rather than the precinct level. While filling out a ranked-choice paper ballot can pose some challenges to voters (Anthony et al. 2019, Coll 2021), paperwork is more accessible than a meeting with complicated rules. Caucus realignment, whereby supporters of a campaign just under 15% provide dramatic scenes for the news media as they urge their neighbors to support their candidate, is not just the source of the supposed quaint charm of the caucus, but lies at the heart of its much greater inaccessibility: it is necessarily synchronous, time-consuming, and inscrutable. First-time caucusgoers are known to leave after the initial straw poll (Winebrenner 1983).

The novel tabulation system introduced by Alaska, Hawaii, Kansas, and Wyoming in the 2020 Democratic presidential nomination process—proportional representation with a 15% threshold, preceded by instant-runoff voting—is, in some sense, a take-away caucus for the "bowling alone" era, in that it replicates the realignment process without the time commitment, communal experience, or inflexibility of the caucus system, with a more intelligible ranked-choice ballot, even if the tabulation remains complex (Horst 2021).

Several scholars have commented on the introduction of ranked-choice voting in the 2020 presidential primary. In their review of the ranked-choice tabulation, Liu et al. (2021) argue that ranked-choice voting can give more representation to minority views in the party. While Elizabeth Warren did not break 15% in any of the ranked-choice elections, which were all held after she withdrew, she did gain substantial support, redistributed from less popular candidates. In the RCV states, her

[8] A century ago, "snap" caucuses could be called by local leaders at inopportune moments for opposing factions (Winebrenner 1983).

score improved, from a first-round range of 4.5% (Wyoming) to 7.1% (Alaska) to a score of 5.58% (Hawaii CD2) to 8.2% (Alaska) just before her elimination. Though in the four entirely ranked-choice elections Warren finished well below 15% threshold, her score did improve substantially in relative terms, between 11.16% (Hawaii CD2) to 28.89% (Wyoming) from the low base with redistributed votes, before her elimination and the transfer of those votes to Biden and Sanders. After imputing these improvements between first- and final-alignment vote counts, Liu et al. identify four state-level traditional proportional representation contests in which Warren likely would have received delegates: California, Oklahoma, the District of Columbia, and Democrats Abroad contests. These delegates, they argue, would have been a win for representation: "Warren showed that her voice as a woman and progressive was appreciated by many Democratic voters in 2020 through their ranked choices. RCV in the future would send more delegates representing less-popular candidates to the national conventions to represent the voices of a diverse electorate."

If anything, though, this redistribution *understates* the breadth of her support. With IRV tabulation, only ballots ranking first a lower-performing candidate like Patrick, Gabbard, Yang, Buttigieg, Steyer, Klobuchar, and Bloomberg would have their second (and perhaps third,...) choices examined. In practice, this pool was disproportionately composed of Bloomberg supporters—a candidate whom Warren ruthlessly attacked for being an "arrogant billionaire" who "calls women 'fat broads' and 'horse-faced lesbians'" (Jamieson 2020). On the other hand, Sanders supporters were inordinately likely to rank Warren second, but in most contests IRV would not access the second choices of those ballots, as Sanders was above 15% in the first alignment.[9] There is no notion of a "marginal delegate quota" in an RCV proportional representation system and so single-transfer voting would be extremely difficult to implement: rounding thresholds depend on the *preferences* of the voters, not their aggregate turnout number. Actually existing

While ranked-choice voting is more accessible than in-person caucuses, Liu et al. (2021) suggest that the increased opportunities for voter expression afforded by the ballot design and tabulation method could have motivated exceptional turnout in these usually sleepy late-season primaries and caucuses. After noting that "[i]n Kansas and Nevada, the turnout increased from five-digit numbers before 2020 to six-digit ones in 2020," they conclude that "to...engage more voters to participate in traditionally low Democratic turnout states, such as Wyoming and Alaska, RCV shows great potential." While they do note the Covid-19 pandemic as a confounding variable, other factors were at play—most notably, the switch between 2016 and 2020 from caucuses to primaries in three of the

[9] While the pundit concepts of "ideological lanes" and policy-driven voting are overstated and many Sanders voters did rank Biden second, Warren consistently performed well among Sanders voters—many of whom were Warren supporters voting tactically—in surveys that examined second preferences (such as (Morning Consult 2020)).

four RCV states and, in the fourth, from an in-person, time-intensive caucus to (due to the pandemic) one with universal absentee voting.

In a special issue of *Politics and Governance* on ranked-choice voting, Richie et al. (2021) make the case for including ranked-choice voting in the presidential nomination process. Citing the 2016 Republican nomination debacle, where Trump won with a high floor and low ceiling among divided opposition (and with no proportional representation!), they suggest that ranked-choice voting can produce consensus nominees. They also look at data from the 2020 Democratic primary elections (using proportional representation!) to find that voters took advantage of the ranked-choice ballot where it existed: "Even though Joe Biden had become the presumptive nominee prior to the first fully ranked-choice primary election, nearly three out of every four voters ranked multiple candidates" (Richie et al. 2021). They argue, moreover, that ranked-choice voting can reduce what they call "wasted" votes: that is, votes for candidates who do not end up receiving delegates, or perhaps who have withdrawn but still win delegates (instant-runoff voting does not prevent the accumulation of delegates for withdrawn candidates). Finally, they suggest that ranked-choice voting, including in this novel system, would create incentives for positive campaigning, promote unity around "coalitional candidates"—those who can appeal enough to a broad swath of the party's voters to secure many backup votes (at least from the supporters candidates who cannot clear 15% threshold)—and could even reduce tactical voting. "Voters could express their true preferences at the ballot box rather than engaging in 'strategic' voting based on trying to make the most of a single choice."

It is worth taking a moment to consider how the popular notion of a "wasted" vote[10] does not transfer neatly to single contests in a lengthy nomination process. In a media environment where small changes in polling results that are essentially the consequence of sampling error and methodology can increase a candidate's coverage in the media more than a policy rollout, and thereby materially affect the candidate's position in the race, raw vote counts are not to be discounted. Even when they are situated well below the delegate-accumulation threshold, overperformance relative to polling results can signal momentum, or Joementum. As Joe Lieberman said while votes were being counted on the night of the 2004 New Hampshire presidential primary, "we are in a three-way split decision for third place."

Moreover, it is not clear at all that this novel system would reduce tactical voting. In the days before 2020's Super Tuesday contests, for instance, it was clear to many progressive organizations, based on polling and fundraising numbers and the increased ease with which Biden would clear 15% in contests due to consolidation in the moderate wing of the party, that Elizabeth Warren did not

[10] Obligatory reminder from a voter registration volunteer: every vote counts—even votes for candidates who do not win! No vote that is successfully cast (i.e., not lost to spoilage) is wasted! The media do track the national popular vote and not just delegate counts.

have a realistic path to the nomination (The Nation 2020). Endorsements of Bernie Sanders and calls for Elizabeth Warren to publicly commit to pool her delegates with Sanders's[11] proliferated among pundits and institutions of the progressive wing of the party (Withnall 2020). Many voters responded to these pitches and voted tactically.

Under ranked-choice voting, it is easy to imagine the majority of these progressive institutions calling on a voter who identifies with them and who prefers Warren to Sanders to tactically rank Sanders ahead of Warren on a ranked-choice ballot, to promote consolidation and help Sanders—the only competitive progressive candidate—keep pace with Biden. Others, imagining a contested convention and delegate pooling, might impel voters to rank Warren first, as her lower polling numbers would make clearing 15% more difficult, and Sanders second. Thus, Warren voters hoping for consolidation would be induced to vote Sanders, and Sanders supporters subscribing to the theory that the two candidates with different demographic appeal could pool their delegates to advance a progressive policy would tactically vote Warren, most at risk of not clearing 15%, but with the assurance that that vote would transfer to Sanders in the case that Warren does not clear 15%.[12] Of course, behind such calls is an assumption that the voting system would translate votes for these candidates into delegates in a legible, non-paradoxical fashion—which, as we shall see section 2, is not the case.

In that same issue of *Politics and Governance*, Coll (2021) uses a national survey sample of 1000 likely Democratic Party presidential primary voters to consider the effects of ranked-choice voting on the election. They find that older, less engaged, and more ideologically conservative voters reported more difficult to rank all candidates, but that undervoting is low (in line with what is seen in the literature on real ranked-choice elections—12%) and (unlike much of the literature) not particularly associated with any other studied factor besides reported difficulty with ranking candidates and age (with younger, not older voters, more likely to under-vote). The survey has some flaws, such as the inclusion of only five candidates at a time (November, 2019) when there were more than a dozen in the race. While the concept of "ideological lanes" does not explain voter behavior well, it might explain these survey results. While voters in the "very liberal" lane could rank all of their preferred candidates who were still in the race (namely, Sanders and Warren), liberal-lane voters had just one candidate (Harris) and moderate/conservative voters had just two (Biden and Buttigieg) of many. While the paper does reflect on the complexity of the ballot for the voter, the survey methodology can only conjure up hypothetical ballots, not DSP-compliant ballots designed by party volunteers.

[11] Some argued that, while both candidates occupied a progressive "ideological lane," Sanders' and Warren's combined support was greater than their individual support due to their different demographic "bases" and—provided both exceeded the 15% threshold in most races, which seemed unlikely—both could stay in.

[12] Even in small contests, attaining 15% brings at least two delegates. Marginal votes for a candidate who already clears 15% might bring just one delegate. However, the behavior of the novel system is complex, and this tactical strategy could easily backfire.

While volunteer election administrators for political parties were not surveyed about their experience with ranked-choice voting in 2020, there is some evidence that ranked-choice voting is unpopular with paid election administrators because it is difficult to implement (Anthony et al. 2019). In Democratic Party primaries, which use proportional representation, there is not the upside of preventing spoilers that motivated Mainers to adopt their system after Paul LePage won the governorship twice without securing a majority due to vote splitting among opponents of LePage. With less potential upside, it is worth investigating the downside: party volunteers have to implement a switch to a new tabulation system without state resources to pay for the extra time it takes to administer elections and count votes. This change can increase barriers to participation in party leadership, foment distrust in the party, and precipitate burnout among party volunteers.

Other publications (De Wolff et al. 2021) recommend ranked-choice voting for the presidential nomination race, essentially motivated by Trump's 2016 early primary wins with plurality but not majority support in a party whose nomination process involves many contests with a winner-take-all (rather than proportional) allocation of delegates.

Only Jones et al. (2023) note that the IRV patch for the 15% threshold introduces what is, in effect, an unstudied electoral system that combines aspects of ranked-choice voting and proportional representation in a novel way. While some ranked-choice systems are sometimes called proportional representation, this appellation is used to distinguish the use of single-transferable vote to elect the Australian House of Representatives (with single-member districts) from the single-transferable vote used to elect the Australian Senate (with multi-member districts and the possibility of voting for grouped candidates under party lists). It does not pertain to, for instance, the allocation of delegates to candidates via Hamilton's largest-remainder method, which is the system used by the Democratic Party. In principle, the Democratic Party could adopt such a system: candidates could submit lists of delegates and voters could rank a set of grouped delegates next with a single stroke of a pen in favor of the candidate to which the delegates are pledge. In fact, FairVote advocates for this very system. Unfortunately, their path to securing its adoption passes through the introduction of a novel, unintelligible system more consistent with party rules.

While it is true that ranked-choice voting can be integrated into a proportional representation system, it is important to see whether the resulting system has an increased tendency to generate paradoxical, non-leader, and non-monotonic outcomes and consider other factors that affect its accessibility and intelligibility.

2. The Novel Electoral System Is Susceptible to the No-Show Paradox

Suppose a pool of voters has the complete preferences given in table 1. (~~Teenagers on Election Twitter~~ Astute observers will notice that the raw vote counts and delegates available—but not the

Table 1 Ballots (without T turnout operation).

ballot type	ballots	counts	percentages
1	A > (T > B > C)	23139	57.9%
2	B > (T > C > A)	9059	22.7%
3	C > (A > T > B)	5730	14.3%
4	T > C > (B > A)	2056	5.1%

Four distinct preference sets are expressed in this hypothetical ranked-choice election. Only A and B are listed as the first choice on at least 15% of ballots and they, under a traditional one-candidate-per-ballot PR election with the Democratic Party's 15% threshold, would receive all the delegates. With instant-runoff voting, the ballots of type 4 redistribute their vote from T to C, bringing C above the 15% threshold. Parentheses are placed around the preferences not considered in this example.

Table 2 Tabulation (without T turnout operation).

candidate	PR share	PR raw	PR apportioned	PR+IRV share	PR+IRV raw	PR+IRV apportioned
A	71.9%	9.34	9	57.9%	7.52	7
B	28.1%	3.66	4	22.7%	2.95	3
C	0%	0	0	19.5%	2.53	3
T	0%	0	0	0%	0	0

From left: the candidates, their shares of the delegate-accumulating vote (without IRV), the corresponding raw fraction of the 13 delegates, and the delegate counts after Hamilton apportionment of the surplus. The final three columns display the same numbers but after IRV, so that the votes of ballot type 4 are first transferred to C.

(unexpressed) preferences—were taken from the 2020 Democrats Abroad Global Presidential Primary, where there were 23139 unspoiled votes for Bernie Sanders, 9059 for Joseph R. Biden, 5730 for Elizabeth Warren, and 2056 for everyone else.)

If the voters in this pool are invited to express only their first choice on a ballot, the traditional Democratic Party proportional representation system in a contest with 13 delegates at stake (as is the case with the Democrats Abroad Global Presidential Primary) will allocate 9 delegates to candidate A and 4 to candidate B (see table 2, columns 2-4). If, however, IRV is first used to eliminate candidates until all those remaining attain the 15% threshold, A will receive 7 delegates; B, 3; and C, 3 as well (table 2, columns 5-7). In this case, IRV brings C above the threshold and differences in support between B and C and A and B are each flattened. The only second choices that were considered were those of voters who ranked T first.

So far, this example is similar to the case study of (Liu et al. 2021), who imputed Warren's share increase using data from later ranked-choice contests to argue that Warren likely would have cleared 15% in the Democrats Abroad Global Presidential Primary and received about 3 delegates. This sample tabulation assumes, of course, that the change in tabulation method does not affect voter preference expression. It is certainly possible that the substitution of some tactical votes for A or B for votes for C would accompany this change in tabulation system. In this example, we have selected downballot preferences rather favorable to T. In fact, we can alter the ballots of type 3 so

Table 3 **Ballots (with 3673-vote T turnout operation).**

ballot type	ballots	counts	percentages
1	A > (T > B > C)	23139	53.0%
2	B > (T > C > A)	9059	20.8%
3	C > (A > T > B)	5730	13.1%
4	T > C > (B > A)	5729	13.1%

The same scenario as in table 1, except T executed a successful turnout operation that led to an additional 3673 type 4 ballots cast.

Table 4 **Tabulation (with 3673-vote T turnout operation).**

candidate	PR share	PR raw	PR apportioned	PR+IRV share	PR+IRV raw	PR+IRV apportioned
A	71.9%	9.34	9	53.0%	6.89	7
B	28.1%	3.66	4	20.8%	2.70	3
C	0%	0	0	26.2%	3.41	3
T	0%	0	0	0%	0	0

The same as table 2, except with the 3673 additional type 4 ballots.

that T is the second preference, and T will still receive zero delegates under the novel IRV-and-PR system, despite being ranked first by 5.1% of voters and second by the remaining 94.9%. Since the IRV tabulation will only examine the second choices of ballots of type 4, i.e., of T voters, the broad acceptability of T compared to the alternatives goes completely unnoticed by the tabulation method.

In this example, however, it is not the selective scrutiny of second choices that we are interested in, but rather the *no-show paradox*, a particular sort of non-monotonic phenomenon whereby the participation criterion is violated: additional ballots of a type j cast can, all else equal, produce a outcome that is strictly worse at respecting the preferences ballot j conveys. We are aware of no definition in the literature that applies to the particular case of proportional representation with ranked-choice ballots; we supply one that is more mathematically rigorous in definition 5. As we tweak the vote tallies in table 1, we do not require a mathematically rigorous definition of what "strictly worse" means to observe that the this novel tabulation system is susceptible to the no-show paradox. All reasonable definitions of "strictly worse" will agree in this case.

Let us increase the share of ballots of type 4. Suppose candidate T mounts a turnout operation that yields an additional 3673 ballots of type 4 (see table 3). This would not be enough to change the final allocation of delegates under either the traditional proportional representation system or the novel electoral system with threshold-aware IRV (see table 4).

Now suppose an additional two ballots of type 4 are cast.[13] Now C is eliminated before T, and all ballots ranking C first (i.e., those of type 3) are transferred to their second candidate (i.e., A). Since T remains below 15%, the ballots of type 4 are redistributed again—not to C, who was eliminated,

[13] If ties are broken using lexicographical order, the same result holds with just a single marginal ballot, but we wish to elide the question of tie-breaking here.

Table 5 **Ballots (with 3675-vote T turnout operation).**

ballot type	ballots	counts	percentages
1	A > (T > B > C)	23139	53.0%
2	B > (T > C > A)	9059	20.7%
3	C > A > (T > B)	5730	13.1%
4	T > C > (B > A)	5731	13.1%

The same scenario as in table 1, except T executed a successful turnout operation that led to an additional 3675 type 4 ballots cast.

Table 6 **Tabulation (with 3675-vote T turnout operation).**

candidate	PR share	PR raw	PR apportioned	PR+IRV share	PR+IRV raw	PR+IRV apportioned
A	71.9%	9.34	9	66.1%	8.60	9
B	28.1%	3.66	4	33.9%	4.40	4
C	0%	0	0	0%	0	0
T	0%	0	0	0%	0	0

The same as table 2, except with the 3675 additional type 4 ballots.

but to B. Thus, by adding two ballots of type 4, the second choice it expresses (C) loses 3 candidates and the third and fourth choices (B and A) gain 1 and 2 delegates, respectively. Had these ballots of type 4 not ranked B and A, they would have been exhausted upon T's elimination and the final result even worse for them: 10 delegates for their fourth choice A, and 3 for their third choice B.

In fact, we can raise the total number of marginal ballots of type 4 by a further 1385 and the result will still be the same: adding 4700 T > C (> B > A) ballots from the base of 2056 to 6756 makes these voters *worse off*.

Of course, T voters are not a bloc. Voting behavior downballot is (usually) not "random,"[14] though it is certainly difficult to model. For instance, in FairVote's ranked-choice poll of the 2020 presidential field, a lot of Bloomberg voters had Sanders ranked ahead of Biden. Nevertheless, it is useful to use random models of voting behavior, both as a benchmark and to compare how they compare at the opposite extreme from the one considered in this section—when voter behavior is not aligned but completely uncorrelated.

In the next section, we consider a three-candidate field. It is certainly on the small side compared to 2020. But most fields are not two-debate-stage large; 2020 was a bit of an outlier. The current 2024 Democratic presidential primary field has three "major" candidates appearing in opinion polls. And in the last two competitive Democratic primaries there were, in effect, only three candidates of note by the night of the Iowa caucuses.

In general, alignment of preferences decreases and the chance that a Condorcet winner cannot be determined increases as the field gets larger. On the other hand, the 15% threshold becomes more difficult to attain.

[14] That talking to voters often leads to bewilderment is not in dispute. But massive traffic spikes are probably not observed by the admins of `random.org` on Election Day.

3. Experiments: Dirichlet Model, Three Candidates

We begin by introducing the random election model we use in our simulations.

Definition 1 (Dirichlet Election Model). Suppose we have an election with n voters and d possible ballots expressing complete, linear preferences (i.e., $d=m!$ with m candidates), enumerated, say, in lexicographical order. In an *impartial culture* election, a sequence of ballots $(b_1,b_2,\ldots,b_n)$ is chosen with each $b_p \sim \mathrm{Unif}(\{1,\ldots,d\})$ selected independently for each voter $p\in\{1,\ldots,n\}$. The vector of tallies $x\in\mathbb{R}^d$ of each of the d ballot types—i.e., $x_i=\sum_{p=1}^n \mathbb{1}_{b_p=i}$—is distributed as follows:

$$p(x)=\frac{n}{d^n\cdot x_1!\cdot\ldots\cdot x_d!}.$$

In an *impartial anonymous culture* election, on the other hand, the voters are not identified, and the result is treated as a "bag of ballots." The election results have probability mass function

$$p(x)=\frac{n!(d-1)!}{(n+d-1)!};$$

this, notably, is uniform for all valid tally vectors $x\in\mathbb{R}^d$.

Both situations are instances of the Pólya-Eggenberger distribution (Berg 1985), whose probability mass function is given by

$$p(x;\alpha,A_1,\ldots,A_d)=\frac{n!\left(\sum_{j=1}^d A_j^{[x_j,\alpha]}\right)}{A^{[n,\alpha]}}$$

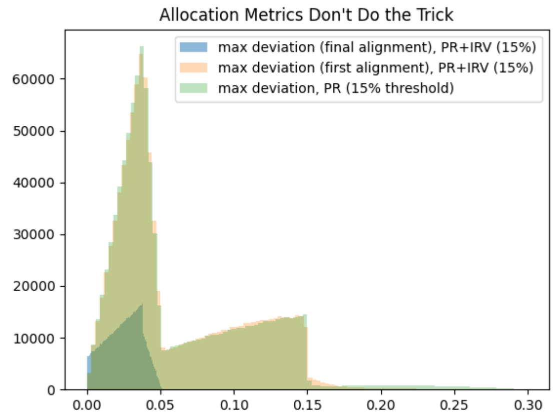
and can be generated by reasoning with a Pólya urn. In both cases, the urn begins with one example of each type of ballot ($A_j=1$ for $j=1,\ldots,d$ and $A=\sum_{j=1}^d A_j=d$). In an impartial culture election, votes corresponds to drawing from the urn with simple replacement (draw a ballot of type b_p, place one ballot of type b_p back in the urn, i.e., the replacement factor $\alpha=0$); hence, voters act independently from one another. In the impartial anonymous culture, a voter who draws ballot b_p places two ballots of type b_p back in the urn (so that $\alpha=1$), inducing some dependence between voters.

Now we take the limiting distribution of the anonymous impartial culture model as $n\to\infty$. Consider the vector $y=x/n$ of vote shares of each type of ballot, so that $y\geq 0$ and $\sum_{i=1}^d y_i=1$. Using the generalized Stirling numbers to replace the rising factorials in the Pólya-Eggenberger distribution, one can derive, supposing $\alpha\neq 0$, a CDF for any vector $y=(y_1,\ldots,y_d)$ in the $(d-1)$-simplex $S=\{y_1e_1+\ldots+y_de_d \mid \sum_{j=1}^d y_j=1 \text{ and } y_j\geq 0 \text{ for all } j\in\{1,\ldots,d\}\}$ of valid vote shares (Berg and Lepelley 1992, Lepelley and Valognes 2003):

$$f(y;\alpha=1,A_1=\ldots=A_d=1)=\frac{\Gamma\left(\sum_{j=1}^d \frac{A_j}{\alpha}\right)}{\prod_{j=1}^d \Gamma(\frac{A_j}{\alpha})}\prod_{j=1}^d y_j^{\frac{A_j}{\alpha}-1}=\Gamma(d)=(d-1)!$$

In other words, $y\sim \mathrm{Dir}(\mathbf{1}_d)$, where $\mathbf{1}_d$ is the length-d ones vector. Thus, a Dirichlet distribution approximates the vote shares of an impartial anonymous culture election with many voters. The

Figure 1 The First- and Final-Alignment Maximum Deviation



probability that, under this model of random elections, a paradox occurs is proportional to its area in the simplex S. Dirichlet distributions with a different vector of initial urn composition $A \neq \mathbf{1}_d$ are used to produce dependent differently distributed models for a wide variety of elections systems (Rolland et al. 2022). Unless stated otherwise, we consider a Dirichlet election to be a Dirichlet model approximation of an impartial anonymous culture election.

We will simulate voter preferences in random elections according to the Dirichlet model. Traditional metrics of apportionment fairness—such as the maximum deviation between a candidate's (first- or final-alignment) vote share and delegate share—are not especially well-suited to understanding the functioning of the novel system. We see in figure 1 that IRV reallocates votes so that the final alignment vote-shares produce a delegate allocation with lower maximum deviation, i.e., with lower maximum absolute discrepancy between the share of votes and share of delegates a candidate receives. But since these are reallocated votes, fairness is difficult to assess. The maximum deviation of the final-alignment simply measures the fairness of the Hamilton apportionment, not the overall system.

One useful starting point is a generalization of the Borda effect.

Definition 2 (Generalized Borda Effect/Generalized Non-Leader Election). The Borda effect occurs in a single-winner election when candidate A wins the election but a majority of voters prefer some other candidate B to A Kamwa (2019).

We say that any delegate apportionment exhibits the Borda effect if a candidate A gets more delegates than candidate B while a majority of voters prefer candidate B to candidate A.

Since the *leader criterion* in traditional proportional representation systems is violated whenever the candidate who receives the most votes receives strictly fewer delegates than some other candidate, we will say that a delegate apportionment system with ballots expressing voters' ranked preferences is *generalized non-leader*, or non-leader as shorthand, whenever there is some candidate A who receives

Table 7 Non-leader outcomes observed in 1,000,000 Dirichlet elections with 15% threshold, awarding 13 delegates

tabulation type	non-leader outcome prevalence
proportional representation (ordinary)	N/A
proportional representation (complete preferences given but ignored)	9.17%
winner-take-all IRV	39.57%
novel system: IRV, then Hamilton apportionment	39.02%

Inserting IRV into a proprotional representation system introduces Borda effects that do not exist (within a single contest). Examining complete preferences does reduce compared to a hypothetical situation of providing voters with ranked-choice ballots only to ignore all but the candidate ranked first. But only in 0.55% of Democrats Abroad-like elections! And, as we will see in 3, the worst such instances, in which the candidates involved in the Borda effect are separated by *many* delegates, increase under the novel tabulation system.

more delegates than candidate B even though a majority of voters prefer candidate B to candidate A—that is, whenever the Borda effect is exhibited.

In table 7, we simulate 1,000,000 elections with three candidates according to the Dirichlet model. All voters have complete preferences among the three candidates. In an IRV election with all delegates going to the winner, there is a Borda effect in 9.17% of the simulated elections. In a traditional proportional representation system with a 15% threshold, where voters express their complete preferences on the ballot but the tabulation only examines the top preference, the Borda effect is exhibited in 39.57% of the elections. (If voters do not express their complete preferences on the ballot, there is insufficient information to identify any Borda effect.) With the novel tabulation system, instant-runoff voting placed before the Hamilton allocation reduces the share of generalized non-leader elections falls slightly, to 39.02%.

Certainly, non-leader elections are undesirable.[15] But not all non-leader elections are equally upsetting. There can be degrees of non-leaderness, for instance, whereby a candidate B can receive *many fewer* delegates than some candidate A despite being preferred to A on a majority (large majority, even) of ballots. And, as the name leader criterion implies, people tend to more worried that the top delegate winner is in the right place, and are less concerned about the Borda effect it if arises among minor candidates.[16]

Moreover, there are many other undesirable properties of electoral systems besides the Borda effect. For instance, we may also wish to avoid non-monotonic outcomes, whereby a small surge in voters casting ballots of type j makes the outcome *worse* for these voters, or at least for the preferences they express. While IRV, in assigning one winner, reduces the opportunities for the Borda effect to emerge, there are good reasons to choose a proportional representation system for the nomination process over winner-take-all primaries: minority factions will be better represented at the conventions, and the race will tend to be competitive longer, testing campaign strength beyond Super Tuesday.

[15] At least, to those who do not spend their days posting “actually, it’s a Republic not a democracy” to Twitter, or using Dave’s Redistricting App to draw the most racist maps possible.

[16] That is, #MayorCheat would probably not have been as popular a hashtag if the violation of the leader criterion had been between the third- and fourth-place finishers, not those first and second place.

While it is difficult to say any one delegate allotment "optimal" with respect to the preferences a ballot expresses without access to a lot of unavailable information, we can provide definitions of undesirable phenomena and quantify their prevalence in random elections. In the rest of this section, we consider susceptibility to the no-show paradox and prevalence of strongly non-leader outcomes.

3.1. Quantifying Susceptibility to the No-Show Paradox

In this section, we introduce a generalization of the definition of the no-show paradox, which allows it to be applied to the novel PR-IRV hybrid system or to other non-standard PR systems susceptible to non-monotonic outcomes.

3.1.1. Defining the No-Show Paradox in Proportional Representation Systems The no-show paradox does not have a standard definition for proportional representation systems. We take majorization—a concept suffused across the terrain of discrete mathematics, algebra, and convex analysis—as inspiration a natural generalization of the paradox, applicable to PR.

Definition 3 (Majorization). Let x and y be two vectors in $\mathbb{R}^n$. We say that *y majorizes x* (written, $y \succ x$) if and only if the following conditions are satisified:

1. for $k = 1, \dots, n$, $\sum_{i=1}^{k} [y]_i \geq \sum_{i=1}^{k} [x]_i$; and
2. $\sum_{i=n}^{k} [y]_i = \sum_{i=1}^{n} [x]_i$.

Here $[x]_i$ selects the ith-largest element of x.

Remark 1. Without the second item, we can say y weakly majorizes x (often written $y \succ_w x$). If x and y are, say, delegate allocations, the second item will always hold because each contest awards a fixed number of delegates.

Remark 2. Note that $(0,2)$ majorizes $(2,0)$ and vice versa, but $(0,2) \neq (2,0)$; in other words, majorization is not anti-symmetric. Thus, majorization is a *preorder*. With respect to delegate counts, majorization can compare how evenly the delegates are distributed across candidates, although it cannot distinguish between two identically distributed, but different, allocations. Both $(2,0)$ and $(0,2)$ majorize $(1,1)$ because these allocations are each, in some sense, "at least as unequal" as $(1,1)$. A winner-take-all allotment majorizes every other outcome, including the other winner-take-all results.

Majorization has many of the properties we desire, including a close connection to directed graphs and the study of utility-improving transfers. However, we wish to preserve a notion of order within the vector: it matters for the voter's utility *to whom* the delegates are allocated, not just how evenly they are distributed. Thus, we introduce a closely related definition, which we will use to define the PR no-show paradox.

Definition 4 (Preference). Let x and y be two vectors in $\mathbb{R}^n$. We say that x is preferable to y (written, $x \succ_p y$) if and only if the following conditions are satisified:

1. for $k = 1, \ldots, n$, $\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$; and
2. $\sum_{i=n}^{k} x_i = \sum_{i=1}^{n} y_i$.

Here x_i selects the ith element of x, i.e., $x_i = e_i^T x$, where e_i is the ith standard basis vector in $\mathbb{R}^n$.

Remark 3. Since preferability takes into account the order of elements in the vector, it is anti-symmetric, and thus a partial order.

Remark 4. Preferability is distinct from the lexicographical order, which is a total order, as $\mathbb{R}^n$ is a Cartesian product of totally ordered sets. For instance, we can say that, with respect to the reverse lexicographical order $>_{lex}$, $(1,0,1) >_{lex} (0,2,0)$, but neither distribution of two delegates is *preferable* to the other: $1 > 0$ and yet $1+0 < 0+2$.

We are now ready to articulate our definition of the no-show paradox.

Definition 5 (No-Show Paradox for PR with Ranked Preferences). Suppose n candidates are ordered from 1 to n and delegate counts are given as vectors $v \in \mathbb{R}^n$, where the ith element of each delegate count, v_i, gives the number of delegates apportioned to candidate i. We say the *no-show paradox* occurs in a proportional representation system if the addition of ballots of type j, which rank the n candidates $c_{j_1} > \ldots > c_{j_n}$, transforms the delegate allocation from x to y and $\text{sort}(x,j) \succ_p \text{sort}(y,j)$, where the function sort reorders the delegate allocations in v according to the candidate preferences expressed on the ballot of type j: i.e., for all vectors $v \in \mathbb{R}^n$ and permutations $j \in S_n$, $\text{sort}(v,j) = (v_{j_1}, v_{j_2}, \ldots, v_{j_n})$.

It is easy to see that this definition generalizes the one used for single-winner outcomes.

Example 1. Suppose a winner-take-all, d-delegate contest grants d delegates to candidate B. After the inclusion of marginal ballots of preference B > A > C, the winner changes to A. Since the initial delegate assignment, sorted by the preference B > A > C, $x = (d,0,0)$ is preferable to the final assignment $y = (0,d,0)$, we say the no-show paradox is present.

Remark 5. The poset $(\mathbb{R}^n, \succ_p)$ is a Riesz space. We can write $x \vee y = (\max(x_1, y_1), \max(x_1 + x_2, y_1 + y_2), \ldots, \max(x_1 + \ldots, x_n, y_1 + \ldots, y_n))$.

We will now establish a felicitous property of our notion of preferable delegate allocations.

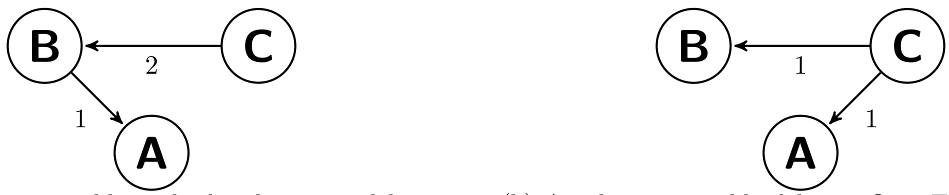
Proposition 1 **(Preferable allocations move delegates up-preference)**. *An allocation y is preferable to x if and only if the allocation x can be transformed into y by a sequence of up-preference transfers.*

Proof: Deferred until the appendix.

Remark 6. This result is closely related to digraph realization problems in majorization theory and the flow-network proof is rather similar to proofs used in that context (see, for instance, chapter 6, theorem 1 and 4 of (Berge 1970).)

Figure 2 ***Preferable* changes in delegate allocation can be explained by up-preference delegate flows.**

In this example, we suppose ballots of type j have preferences $B > A > C$, and initially B receives 3 delegates; A, 3; and C, 0. After injecting ballots of type j into the pool, tabulation and apportionment are re-run, and all candidates end up with two delegates. Since $(3,3,0) \succ_p (2,2,2)$—i.e., when delegate counts are sorted by j's preferences, the initial apportionment is preferable to the final apportionment—we can, by proposition 1, find a finite set of up-preference transfers to transform $(2,2,2)$ into $(3,3,0)$, as in subfigure (b), even though mixed transfers like those in subfigure (a) are also possible. Thus, the no-show paradox has occured.



(a) One flow compatible with the change in delegate allocation. Delegates transferred from C to B flow up-preference; those from B to A, down-preference.

(b) Another compatible delegate flow. This transfer is entirely up-preference, which means the change was preferable with respect to ballot j.

3.1.2. Experiments Direct simulations of the no-show paradox, in which a single marginal ballot moves the election outcome away from that ballot's expressed preferences, are a bit hard to formulate. First, the phenomenon is rare, requiring large numbers of elections to simulate. Second, the model of random elections we use hides from users the election's size. Thus, we apply an *impulse* δ to a particular ballot type—i.e., we boost its share (and reduce the others accordingly)—to simulate marginal turnout. As algorithm 2 indicates, for each simulated election, we apply the impulse to each of the six possible ballot types and note a no-show outcome if at least one of those six impulses produces the paradoxical outcome. That is, we take the ballot shares $x \sim \mathrm{Dir}(\mathbf{1}_6)$ and perform tabulation. Then we add δe_i for $i = 1, \ldots, 6$, where e_i is the ith standard basis function of $\mathbb{R}^6$, and perform tabulation again. If the first outcome is distinct form the second and preferable, with respect to the preferences exhibited by ballots of type i, then we consider the outcome an instance of the no-show paradox. Algorithms 1-2 give more detail.

Figure 3 compares the prevalence of the no-show paradox between traditional proportional representation and the novel PR+IRV system in a simulation of 1,000,000 impartial anonymous culture (Dirichlet) elections. Nearly 3% of random three-candidate elections would be susceptible to the no-show paradox with an injection of 5% of ballots of least one ballot type, versus 2.5% for proportional representation with complete, untabulated preferences. Figure 4 shows that this result holds if we reduce the threshold to be awarded delegates from 15% to 5%.

Of course, proportional representation is not susceptible to the no-show paradox since outcomes are monotonic: more votes for a candidate will never lead to fewer delegates. However, we can

Algorithm 1: Helper functions.

Function $IRV(x)$**:**
  Use instant-runoff voting to determine a single winner given the vector x of shares of each ballot type;
  return the winner;
Function $PR(x, d, t)$**:**
  Examine the first preferences of the ballots whose frequencies x stores;
  Allocate the d delegates to the candidates who clear share t using Hamilton's largest-remainder method;
  return the vector of delegate counts;
Function $PRIRV(x, d, t)$**:**
  Perform IRV until all candidates remaining clear t;
  Apportion the d delegates among the remaining candidates using Hamilton's method;
  return the vector of delegate counts;
Function $NOSHOW\_IRV(w, w', j)$**:**
  Compare the winners w and w' with respect to the preferences of ballot type j;
  return $w >_j w'$, i.e., the boolean answer to the question, Do ballots of type j rank w ahead of w'?;
Function $NOSHOW\_PR(x, x', j)$**:**
  Compare the delegate counts x and x' with respect to the preferences of the jth ballot type;
  return $((\text{sort}(x, j) \succ_p \text{sort}(x', j)) \wedge (x \neq x')$, i.e., the boolean answer to the question, Do the delegate allocations x and x' differ elementwise anywhere and, if so, is x preferable to x' according to the preferences expressed by ballot type j?

imagine a peculiar form of proportional representation whereby voters fill out ranked-choice ballots but tabulation ignores all but the first choice.

Thus we can ask, if voters supply extra information, is this information processed in a legible way, better reflecting the preferences of the voting body and avoiding paradoxical outcomes. When it comes to the novel PR+IRV electoral system, the answer is, for low-delegate contests, *no*, for adding more information *increases* paradoxical outcomes—not compared to a paradox-free baseline where only first preferences are stated, but compared to a hypothetical situation where voters have complete preferences but no avenue to express them on the ballot.

We give an example of the no-show paradox in tables 8-11. While the addition of 300 (30% of the prior electorate!) A > B > C ballots into the pool leads A to gain delegates under pure PR tabulation, the IRV+PR system keeps A at 9 delegates but these voters' second choice transfers 2 delegates to their third—a outcome over which the A > B > C voters prefer the initial allocation.

Algorithm 2: No-show paradox simulation.

Data: The number of trials $n = 1000000$, the delegate-accumulation threshold $t = 0.15$, the number of delegates $d = 13$, and the impulse δ.

Result: The share of elections susceptible to a no-show paradox at impulse level δ under each system.

```
// Initialize counters:
cnt_irv = 0; cnt_pr = 0; cnt_prirv = 0;
for i = 1,...,n do
    x ~ Dir(1_6); /* Simulated vote shares.                         */
    // Determine outcome before impulse:
    irv_winner ← IRV(x);
    pr_alloc ← PR(x,d,t);
    prirv_alloc ← PRIRV(x,d,t);
    // Initialize flags:
    noshow_irv ← False; noshow_pr ← False; noshow_prirv ← False;
    for j = 1,...,6 do
        x' ← (x + δe_j)/(1 + δ);
        // Apply impulse and test for no-show paradox:
        noshow_irv ← noshow_irv ∨ NOSHOW_IRV(irv_winner, IRV(x'), j);
        noshow_pr ← noshow_pr ∨ NOSHOW_PR(pr_alloc, PR(x',d,t), j);
        noshow_prirv ← noshow_prirv ∨ NOSHOW_PR(prirv_alloc, PRIRV(x',d,t), j);
    end
    // Update counters:
    cnt_irv += noshow_irv; cnt_pr += noshow_pr; cnt_prirv += noshow_prirv;
end
Return cnt_irv/n, cnt_pr/n, cnt_prirv/n;
```

Table 8 **Ballots before injection of 300 ballots of type 1.**

ballot type	ballots	counts	share
1	A > B > C	542	54.2%
2	A > C > B	103	10.3%
3	B > A > C	3	0.3%
4	B > C > A	169	16.9%
5	C > A > B	116	11.6%
6	C > B > A	67	6.7%

All candidates clear 15% on the first round before the injection of 300 votes. Afterward, only A exceeds the threshold before IRV tallying.

Table 9 Tabulation before injection of 300 ballots of type 1.

candidate	PR share	PR raw	PR apportioned	PR+IRV share	PR+IRV raw	PR+IRV apportioned
A	64.5%	8.385	9	64.5%	8.385	9
B	17.2%	2.236	2	17.2%	2.236	2
C	18.3%	2.379	2	18.3%	2.379	2

Both PR and the novel PR+IRV system generate the same $(9, 2, 2)$ delegate split with 13 delegates and a 15% threshold.

Table 10 Ballots after injection of 300 ballots of type 1.

ballot type	ballots	counts	share
1	A > B > C	842	64.8%
2	A > C > B	103	7.9%
3	B > A > C	3	0.2%
4	B > C > A	169	13.0%
5	C > A > B	116	8.9%
6	C > B > A	67	5.2%

All candidates clear 15% on the first round before the injection of 300 votes. Afterward, only A exceeds the threshold before IRV tallying.

Table 11 Tabulation after injection of 300 ballots of type 1.

candidate	PR share	PR raw	PR apportioned	PR+IRV share	PR+IRV raw	PR+IRV apportioned
A	72.7%	13	13	72.9%	9.48	9
B	13.2%	0	0	0%	0	0
C	14.1%	0	0	27.1%	3.52	4

After the injection of 300 ballots of type 1, the new allocation $(13, 0, 0)$ under vanilla PR is preferable for these voters to the prior allocation $(9, 2, 2)$. On the other hand, under the PR+IRV system, these new ballots of type 1 cause two delegates are transferred from B to C, making these voters worse off.

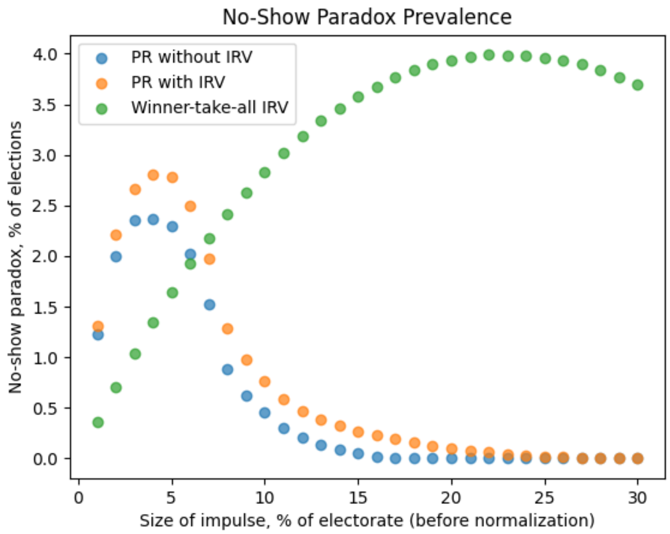
3.2. Quantifying the Prevalence of Non-Leader Outcomes

We will again use the impartial anonymous culture (Dirichlet) model to estimate the prevalence of what we call non-leader outcomes, that is outcomes exhibiting the Borda effect (see 2), in random three-candidate elections under the different voting systems. A proportional representation system—without geographic weighting in the style of the Iowa caucus system—does not have enough information on the ballot to assess the presence of the Borda effect. We assume that the complete preferences always exist and are always expressed on the ballot, but that the proportional representation system only examines the top choice of each ballot. In this way, we can find some non-leader elections.

[18] Democrats Abroad awards delegates in three different global regions (there are no overseas districts), but this is a formality to ensure geographic diversity. The apportionment of delegates to candidates depends only on their global performance, not their performances in these regions.

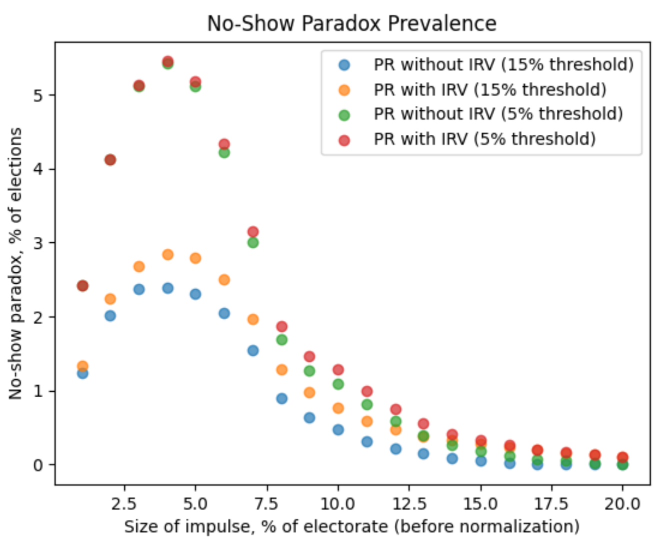
[18] This corresponds to a larger, state-run primary state like Washington, which granted 89 delegates in 2020. 58 of these were awarded at district conventions in each of the 10 congressional districts, which were allocated between 3 and 11 delegates each, and 31 pledged statewide delegates were awarded at the state convention in two separate contests: 12 delegates were conferred in the contest for party leaders and elected officials (these are pledged delegates, not automatic or “super” delegates), and 19 in the at-large contest.

Figure 3   No-show paradox instances in three-candidate elections, with the Dirichlet elections model.



We compare winner-take-all IRV, PR, and the novel PR+IRV system. Note that pure proportional representation is not susceptible to the no-show paradox, so we suppose that there are complete preferences marked on the ballot that proportional representation tabulation (which considers only first choices) ignores. In this example, the 15% threshold is used.

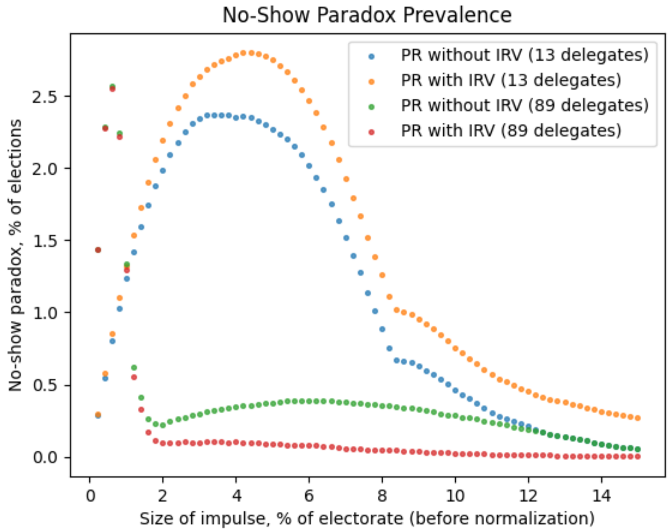
Figure 4   No-show paradox instances in three-candidate elections, with the Dirichlet elections model: comparing thresholds.



Lowering the delegate-accumulation threshold increases the prevalence of random elections susceptible to the no-show paradox but does not change the tendency for the novel electoral system to be more susceptible to no-show paradox than a hypothetical ranked-choice tabulation system that ignores downballot rankings and performs proportional representation on the first choices.

Algorithm 3 details the simulation we conduct to assess the prevalence of non-leader outcomes in random elections under the novel electoral system. In particular, the simulation seeks to estimate

Figure 5 No-show paradox instances in three-candidate elections, with the Dirichlet elections model: comparing delegate hauls.



Increasing the delegate prize of a contest from 13 (the level of Democrats Abroad[17] *and nearly the level of the 2020 ranked-choice contests, which had 14, 15, 24, and 39 delegates) to 89*[18] *allows the IRV patch to use the extra ballot information more effectively, reducing the no-show paradox incidence from the (much lower) baseline. The no-show paradox incidence is clearly greater with the novel system for the 14- and 15-delegate contests; for contests of intermediate size, the difference is negligible for smaller impulses.*

the prevalence of *highly* non-leader outcomes, in which candidate A is ranked above candidate B by a majority of voters, and yet B gets *many* more delegates than A. Figures 6- 7 assay the prevalence of these outcomes with five different contest delegate hauls; figure 8, with two different proportional representation delegate thresholds. While we saw in table 7 that the IRV step can reduce the prevalence of non-leader outcomes slightly in Dirichlet elections compared to a hypothetical proportional representation tabulation system that ignores voters' downrank choices, this novel tabulation is more susceptible to the most non-leader outcomes.

While the failure of the leader criterion was a key factor in complaints about the Iowa caucus results, the notion of a discrepancy was important too, as in the case of Des Moines precinct 80 (Linkletter 2020). Further studies involving new fairness metrics suitable for this novel tabulation system are needed to determine whether its outcomes are any better in typical cases than vanilla proportional representation tabulation that ignores candidates ranked 2 through n. The introduction of non-monotonicity and Borda effects (compared to ordinary proportional representation) and marginal increase in these paradoxical outcomes (compared to ordinary proportional tabulation of ballots that express complete preference) may indeed be a worthwhile price to pay for better outcomes in certain situations.

Algorithm 3: Simulated incidence of non-leader outcomes, by electoral system.

Data: The number of trials $n = 1000000$, the delegate-accumulation threshold $t = 0.15$, the number of delegates $d = 13$, and the impulse δ.

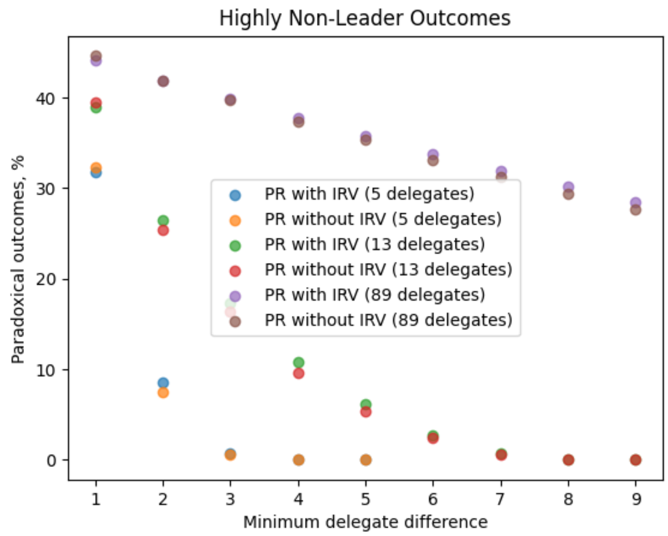
Result: The share of elections with non-leader outcomes at impulse level δ under each system.

```
// Initialize counters:
cnt_irv = 0; cnt_pr = 0; cnt_prirv = 0;
for i = 1, ..., n do
    x ~ Dir(1_6); /* Simulated vote shares. Say x lists vote shares of ballot
       types in lexicographical order of preference.                      */
    // Determine outcome:
    irv_winner ← IRV(x); pr_alloc ← PR(x, d, t); prirv_alloc ← PRIRV(x, d, t);
    vote_shares ← (sum(x[: 2]), sum(x[2 : 4]), sum(x[4 :]));
    // Initialize flags:
    nonlead_irv ← False; nonlead_pr ← False; nonlead_prirv ← False;
    // Count non-leader outcomes:
    for c ∈ {1, 2, 3} \ {irv_winner} do
        nonlead_irv ← nonlead_irv ∨ vote_shares[irv_winner] < vote_shares[c];
    end
    for c = 1, 2, 3 do
        for d ∈ {1, 2, 3} \ {c} do
            nonlead_pr ← nonlead_pr ∨ (vote_shares[d] > vote_shares[c] ∧ pr_alloc[c] >
             pr_alloc[d]);
            nonlead_prirv ← nonlead_prirv ∨ (vote_shares[d] >
             vote_shares[c] ∧ prirv_alloc[c] > prirv_alloc[d]);
        end
    end
    // Update counters:
    cnt_irv += nonlead_irv; cnt_pr += nonlead_pr; cnt_prirv += nonlead_prirv;
end
Return cnt_irv/n, cnt_pr/n, cnt_prirv/n;
```

4. Interactions with Geographic Weighting

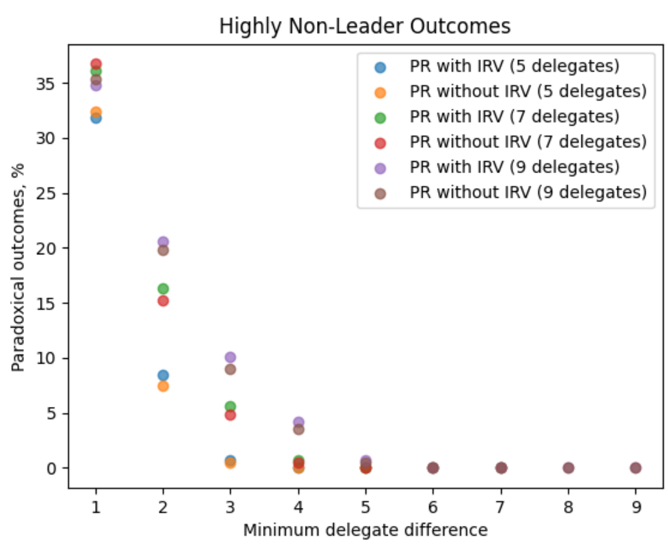
In many states, the Democratic Party's presidential primary is susceptible to non-leader outcomes due to aggregation effects Jones et al. (2019), as we saw in Iowa in 2020 Jones et al. (2023). This is largely due to geographic weighting of votes in district-level contests, the sizes of the delegate prizes of which are decided before the first vote is even cast. Since the numbers released by the

Figure 6 Delegate discrepancies in non-leader outcomes, with different contest delegate awards.



While contests with more delegates were less susceptible to the no-show paradox, they are more susceptible to highly *non-leader outcomes, whereby candidate A wins* many more *(raw, not as a share of the total) delegates than candidate B when a majority of voters prefer B to A. In all cases, using IRV to examine the full preferences on ballots increases the prevalence of highly non-leader outcomes, compared to running vanilla proportional representation after probing only the first choice. Keep in mind that with a delegate prize of 89 and 15% threshold, the minimum delegate award is 13.*

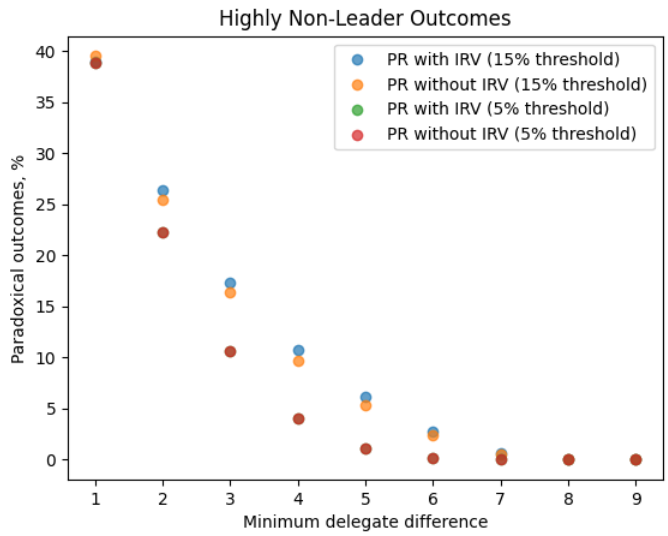
Figure 7 Delegate discrepancies in non-leader outcomes, with different contest delegate awards.



The same as 6, but with smaller delegate numbers, typical of district-level contest awards.

Iowa Democratic Party were not prepared with conscientiousness, let us consider a nearby state with primaries administered by paid professionals, and even processed into a form suitable for the district-level party-run proportional representation contests by state employees. Figure 9 shows how voters

Figure 8 **Delegate discrepancies in non-leader outcomes: comparing thresholds.**



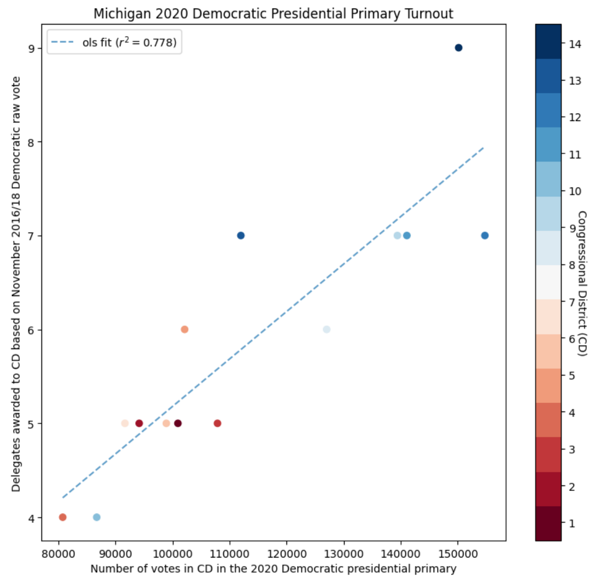
Reducing the threshold for delegate acquisition to 5% reduces the incidence of non-leader outcomes, especially the most severe instances—situations where some candidate A receives many more *delegates than another candidate B while a majority of voters prefer B to A.*

in MI-14 represent 9.458% of the voters but decided 10.976% of the delegates, for a discrepancy of 1.517%. The median absolute discrepancy between share of the vote and share of delegates was 0.467%.

In figure 10, we run an independent three-candidate Dirichlet election in each of the 14 districts, whose contests award 4-9 delegates, and using these results and the districts' populations, award an additional 43 at-large delegates based on the statewide vote. In this situation, districts tend not to be near "tipping points" at the same time, and the use of separate district-level contests tends to reduce the most egregiously non-leader outcomes, compared to a single at-large contest. However, as this model is not realistic, caution must be exercised before applying this result to actual elections. In figures 11-12, we perform the same analysis, but with uniformly sized districts (awarding 4 delegates and 7, respectively) and an at-large delegate pool scaled accordingly.

That the inclusion of a round of IRV tends to slightly reduce the most common form of non-leader outcome (a margin of one delegate) while slightly increasing the prevalence of more egregious—and rare—non-leader outcomes can explain the discrepancy between these results and the results in section 3. With pure proportional representation, district-level contests introduce non-leader outcomes via cascading rounding error and the geographic weighting of votes. Proportional representation tabulation of ranked-choice ballots increases non-leader outcomes involving a single delegate but decreases the prevalence of more pronounced Borda effects. With a Dirichlet model, district-level contests invert this relative difference while greatly reducing the incidence of the most blatant non-leader outcomes.

Figure 9 Delegate discrepancies in non-leader outcomes, with different contest delegate awards



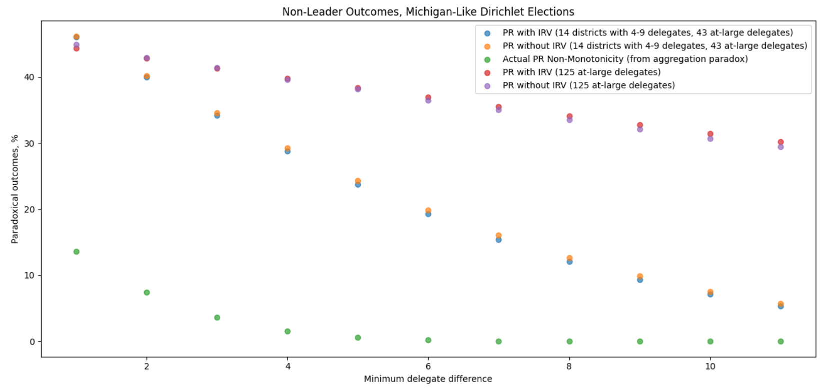
Notice that (the 2011-2021) MI-12 (represented by Debbie Dingell) received 154,762 votes to MI-14's (Brenda Lawrence) 150,164 but only 7 delegates to MI-14's 9. Whereas MI-14 was a Democratic vote sink, stretching from downtown Detroit to Pontiac, in which Republicans benefited slightly from racial depolarization in the Trump era, the heart of MI-12 was in Washtenaw County. MI-12, like MI-3 (Justin Amash), MI-8 (Elissa Slotkin), MI-9 (Andy Levin), and MI-11 (Haley Stevens), was awarded too few delegates because they contained rapidly Democratic-trending communities, in Oakland County and Grand Rapids suburbs; the political composition of these districts in April of 2020 was quite different than in November of 2016 and 2018, when their delegate allocation was determined. The under-allotment in MI-4 (John Moolenaar) likely has a different explanation: while portions of the district in the Tri-Cities region did trend Democratic in the Trump era, the district overall trended toward Trump in both raw and percentage terms. However, many Democrats were activated during this time. Moreover, since Michigan has no party registration, all voters can vote in the Democratic primary provided they do not spoil their ballot by voting in another party's primary. In 2020, the Democratic primary was for many voters the more interesting part of the partisan section of the ballot, as Donald Trump's nomination was a foregone conclusion.

5. Conclusions

Introducing instant-runoff voting in the delegate-selection plan renders the proportional-representation system vulnerable to non-monotonic outcomes and violations of the leader criterion, to both of which it is immune in individual contests (though, as we saw in the 2020 Iowa debacle, aggregation paradoxes can occur when contests are needlessly broken down into sub-contests).

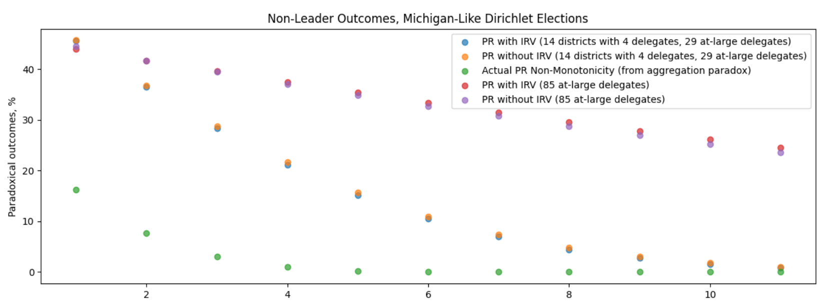
Tables 10-11 give an example where a marginal turnout of 30% of the initial electorate, all casting the same ballot, makes these marginal voters worse off under the novel system, whereas ordinary proportional representation responds appropriately to these new votes. In fact, even when ballots

Figure 10 Non-leader outcomes with district-level and at-large contests



A series of Dirichlet elections were conducted in each of Michigan's 14 congressional districts, with turnout equal to the 2020 turnout, and 43 at-large delegates were attributed based on these results. Compared to a hypothetical system of a single at-large contest based on the statewide total, the most grave non-leader outcomes are less likely, in large part because the minimum delegate award in a single at-large race is 18 delegates. Results under proportional representation tabulation of ranked-choice ballots and the novel tabulation system track each other closely, although their relative positions are inverted, compared to a single at-large race. Non-monotonic outcomes are much more prevalent than under proportional representation with a single-choice ballot.

Figure 11 Non-leader outcomes with district-level and at-large contests

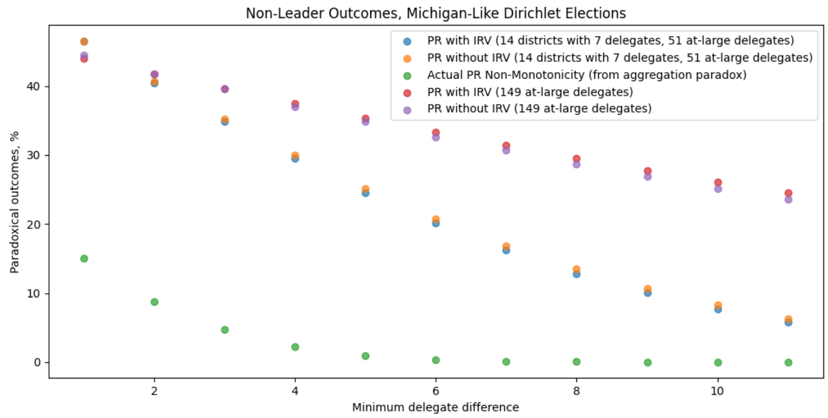


The same as 10, except each district has 4 delegates and the at-large delegate award is scaled accordingly. District sizes are sampled with replacement from the two district sizes awarded 4 delegates in Michigan's 2020 primary—that is, 86,722 and 80,762.

with complete preferences are cast, vanilla proportional representation, which ignores all markings on ballots beyond the top-ranked candidate, is less susceptible to the no-show paradox than this novel PR+IRV system under a random elections model (see figures 3-4)).

The novel system also suffers from what we call non-leader outcomes, i.e., the Borda effect, which we examined in figures 6, 7, and8. Given ballots with complete preferences expressed, this novel tabulation system is no better at translating these preferences into outcomes free of Borda effects than traditional proportional representation tabulation, which considers only the first choice, and does, in fact, have a tendency to produce the most grievous non-leader outcomes, in which some

Figure 12 Non-leader outcomes with district-level and at-large contests



The same as 10, except each district has 4 delegates and the at-large delegate award is scaled accordingly. District sizes are sampled with replacement from the four district sizes awarded 7 delegates in Michigan's 2020 primary—that is, 86,722 and 80,762.

candidate A gets apportioned many more delegates than candidate B when a majority of ballots express a preference of B to A. We find that even with a Dirichlet random model, generalized Borda effects are present in nearly half of elections.

Instant-runoff voting proponents like to dismiss concerns about the Borda effect and non-monotonicity as "academic" and unlikely to occur in practice. Indeed, the requisite foresight, discipline, and coordination needed to adequately profit from monotonicity paradoxes are hard to acquire. But in the social media era, where election math fuels viral conspiracy theories, these are not merely "academic" questions. It is worth seeking to avoid results that can be used to foment disaffection. While it is true, as FairVote argues, that "no group of voters in an RCV election has ever attempted to exploit the possibility of nonmonotonicity for strategic purposes" (FairVote 2023), we have seen recently that election season extends far beyond Election Day. How results are interpreted matters a great deal for the functioning of democracy. Would voters' learning that their votes worked *against* their preferences, after an election, feel as inclined to participate in the future?

A complete analysis of the electoral system must consider the full context in which it is administered—in the case of the novel system, largely by party volunteers—including factors like ballot spoilage rates with volunteer-designed ballots.[19] In the Democrats Abroad Global Presidential Primary, will numerals (with all their global variation) be used to rank candidates? If so, how will ballots

[19] These were already rather high in the Democrats Abroad Global Presidential Primary. Among ballots counted by the remote vote "ballot brigade," of which the author was a member, 12.4% (3516/28349) were spoiled, largely due to failure to provide the information required by the Delegate Selection Plan, including an overseas address. This number is somewhat inflated: all attachments on emails to the ballot-return address were processed as ballots and thus things like email signature attachments were treated as potential ballots and marked as spoiled. Moreover, some voters, experiencing connection difficulties, sent the email twice. Still, nearly 10% of ballots were spoiled at the first Paris voting center before we started asking voters whether they had included each piece of required information before placing their ballot in the urn.

with legibility issues, redundant numerals, and so forth be processed? If checkboxes are used, will this cause the ballot to extend to a second page, potentially leading to email attachment goofs? State parties themselves ought to consider the administrative burden on their core volunteers and observe how this affects the summer and fall campaign season.

Among voters who successfully express their preferences, how confident are they that the math adopted by party volunteers fairly translates the totality of expressed preferences into delegate allocations. That voter behavior is difficult to model, with expressed preferences that can seem perplexing or even arbitrary, does not meant that outcomes can seem perplexing or even arbitrary. Political scientists should not dismiss paradoxical outcomes without first confirming that voters do not find them troubling.

We advise the Democratic Party to make its nominating process simpler and more legible, most notably, by amending all state parties' DSPs so that delegates are apportioned according to statewide performance only. Delegates can still be selected in regional or district conventions while eliminating aggregation paradoxes when delegates are apportioned to these regional conventions based on candidates' statewide performance, as is the case with the DPCA regional conventions.

We also urge organizations such as FairVote, who seek to establish a nomination process using STV in the manner of Australian senate elections, to consider a path toward this change that does not involve a series of incremental kludges that make the system worse than the status quo system, which but for the district-level contests and unusually high threshold, is perfectly satisfactory.

The presence or absence of paradoxes in random elections is far from the only way to assess the performance of an electoral system. But it is an important first step. More work is needed to establish whether the kludge introduced by the Democratic Party achieves its aims of inspiring confidence in the system, reducing unfair outcomes, minimizing tactical voting and voter remorse, and ensuring a better voting experience. As this change continues to be introduced in small, low-salience party primaries, voter feedback that the opportunity to rank multiple candidates is appreciated may color our view of its success. The point of an election is not to give voters the opportunity to speak, but to be heard, and whether that is happening may not be evident in low-stakes contests awarding few delegates. Unless this new system is studied thoroughly before deployment, it might not be evident until its rollout in a high-salience election, where the results are carefully studied and debated, that it fails to hear the voters it invites to speak.

The accessibility of an electoral system is not just about how easy it is to rank candidates and fill out a ballot. It's also about understanding how one's individual vote affected the outcome. The Borda effect and the no-show paradox are much more difficult to understand than an increment of the least-significant digit, which is where one locates one's vote in a plurality system. Any electoral system whose results are uninterpretable is inaccessible. Geographic weighting and IRV lead to illegible outcomes that are far more difficult to explain than a conspiracy theory about rigged math. We do certainly see conspiracy theories proliferate after first-past-the-post elections, and conspiracy theories involve far more than math—including fundamental questions about democracy and who is entitled to equal citizenship—but efforts to simplify the system—including instating the popular vote—would no doubt make elections easier to understand, trust, and participate in.

6. Appendix: Proof of Proposition 1

PROPOSITION 1 **(Preferable allocations move delegates up-preference)**. *An allocation y is preferable to x if and only if the allocation x can be transformed into y by a sequence of up-preference transfers.*

Proof: Since x and y award the same number of delegates,

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i,$$

and we can write

$$0 = \sum_{i=1}^{n}(x_i - y_i) = \sum_{i=1}^{n}(x_i - y_i)_+ - \sum_{i=1}^{n}(y_i - x_i)_+,$$

where $(\cdot)_+$ is the ramp function (often called, as fashion dictates, the ReLU):

$$(x)_+ = \begin{cases} x, & \text{if } x > 0; \\ 0, & \text{otherwise.} \end{cases}$$

Given n candidates, the vector $x \in \mathbb{R}^n$ containing the initial delegate assignment, and a vector $y \in \mathbb{R}^n$ containing the post-ballot-addition delegate assignment, we construct a weighted digraph $G = (V, E)$, where the vertex set $V = \{s, t, d_1, \ldots, d_n, \overline{d}_1, \ldots, \overline{d}_n\}$ and the edge set E, as follows (see figure 13 for an example with 4 candidates):

1. For $i = 1, \ldots, n$, create an edge from s to d_i of capacity $(x_i - y_i)_+$.
2. For $i = 1, \ldots, n$, connect $\overline{d}_i$ to t with capacity $(y_i - x_i)_+$.
3. For each i and j in $\{1, \ldots, n\}$, connect d_i to $\overline{d}_j$ with unlimited (e.g., #delegates) capacity if and only if j is preferred to i in the canonical ordering—that is, if $j < i$.

By construction, the maximum flow in this network (computed, e.g., using the Ford-Fulkerson algorithm) equals $\sum_{i=1}^{n}(x_i - y_i)_+$ if and only if $y \succ_p x$.

To see this, we proceed by induction. Consider the induced subgraph $G' = G \setminus \{d_2, \ldots, d_n\}$. Since d_1 has no outlet, the minimum cut between s and the first layer is $(x_1 - y_1)_+$ (s is in fact separated from t), and this maximum flow is attained if and only if $(x_1 - y_1)_+ = 0$, i.e., if $y_1 \geq x_1$. In this case, the flow is 0, and the first inequality of item 1 of definition 4 is satisfied.

Now suppose for all $j = 1,\ldots,k$, we have established that

$$\sum_{i=1}^{j} y_i \geq \sum_{i=1}^{j} x_i$$

if and only if cutting the flow network $G \setminus \{d_{j+1},\ldots,d_n\}$ at the first layer $\big($i.e., $(\{s\},\{d_1,\ldots,d_j,\overline{d}_1,\ldots,\overline{d}_j,t\})\big)$ produces a minimum cut from s to t. That is, our induction hypothesis assumes the equivalences between the first j inequalities in item 1 of definition 4 and the occurrence of a minimum cut between the s layer and the d_i layer on $G \setminus \{d_{j+1},\ldots,d_n\}$, for all $j = 1,\ldots,k$.

Let us add node d_k to the graph and consider the minimum cut through $G \setminus \{d_{k+2},\ldots,d_n\}$. Because the middle layer has unlimited capacity, the minimum cut must have value

$$\min\left\{ \underbrace{\sum_{i=2}^{k+1}(x_i - y_i)_+}_{\substack{\text{flow between } s\\ \text{and the } d_i \text{ nodes}}}, \underbrace{\sum_{i=1}^{k}(y_i - x_i)_+}_{\substack{\text{flow between the}\\ \overline{d}_i \text{ nodes and } t}} \right\}$$

since the capacity of the edge $d_i \to \overline{d}_j$ is 0 if $j \geq i$. If $(x_{k+1} - y_{k+1})_+ = 0$, the top layer remains the minimum cut and the inequality

$$\sum_{i=1}^{k+1} y_i \geq \sum_{i=1}^{k+1} x_i$$

holds. In this case, we are done.

We suppose that $\delta = (x_{k+1} - y_{k+1})_+ > 0$. We have added δ units of capacity in the top layer; $\overline{d}_{k+1} \to t$ has capacity $(-\delta)_+ = 0$, but we have but also added links from d_{k+1} to $\overline{d}_1,\ldots,\overline{d}_k$. Let us suppose $\delta_1,\ldots,\delta_k$ units of flow passes through the links $d_{k+1} \to \overline{d}_1,\ldots,d_{k+1} \to \overline{d}_k$, respectively. Thus, new capacity obtained from adding d_{k+1} to the graph can only be sent to t if these new links can absorb this capacity and transmit it through the existing links $\overline{d}_{i_1} \to t,\ldots,\overline{d}_{i_l} \to t$. But this is another way of saying the inequality

$$\sum_{i=1}^{k+1} y_i \geq \sum_{i=1}^{k+1} x_i$$

remains satisfied!

In other words, since lower-preference nodes $d_{k+2},\ldots,d_n$ and $\overline{d}_{k+1},\ldots,\overline{d}_n$ cannot receive any flow, the flow network is at capacity when these δ units of flow pass through d_{k+1} and then $\{\overline{d}_1,\ldots,\overline{d}_k\}$ before passing to t. This is equivalent to saying that there exist nonnegative numbers $\delta_1,\ldots,\delta_k$ such that $\delta = \delta_1 + \ldots + \delta_k$ and

$$\begin{aligned} y_1 &\geq x_1 + \delta_1 \\ &\vdots \\ y_k &\geq x_k + \delta_k. \end{aligned}$$

Summing these k inequalities, we obtain

$$\sum_{i=1}^{k} y_i \geq \sum_{i=1}^{k} x_i + \underbrace{(x_{k+1} - y_{k+1})_+}_{\delta = \sum_{i=1}^{k} \delta_i},$$

or, equivalently, since $(x_{k+1} - y_{k+1})_+ > 0$ by assumption,

$$\sum_{i=1}^{k+1} y_i \geq \sum_{i=1}^{k+1} x_i,$$

which is what we wanted. By construction, adding these δ units of capacity at top layer by adding d_{k+1} raises the maximum flow by all δ units (i.e., cutting off the top layer remains the minimum cut) exactly when the $k+1$st inequality of definition 4 is satisfied.

The result follows from induction. Q.E.D.

Acknowledgments

The author gratefully acknowledges the US Democratic Party for uniting the love of math and distaste for tedious and needlessly complex administrative processes in the sacred institution of procrastination from thesis-writing. Unfortunately, intellectual stimulation is not the point of party business, Jack.

References

Aldrich, J. (2009). The Invisible Primary and Its Effects on Democratic Choice. *PS: Political Science & Politics, 42*(1), 33-38. doi:10.1017/S1049096509090027

Anthony, J., Fried, A., Glover, R., & Kimball, D.C. (2019). Ranked Choice Voting in Maine from the Perspective of Election Authorities. In the Annual meeting of the American Political Science Association, Washington, DC.

Associated Press (2020). "AP decides not to declare Iowa caucus winner after recount." Retrieved July 14, 2023, https://apnews.com/article/fc6777e93b8c50b2fd20e0d31fcc43b3.

Becker, A., and Martina, M. (2020). "Buttigieg narrowly wins Iowa caucuses: state party results." *Reuters*. Retrieved July 14, 2023, https://www.reuters.com/article/us-usa-elections-idUSKBN2002JS

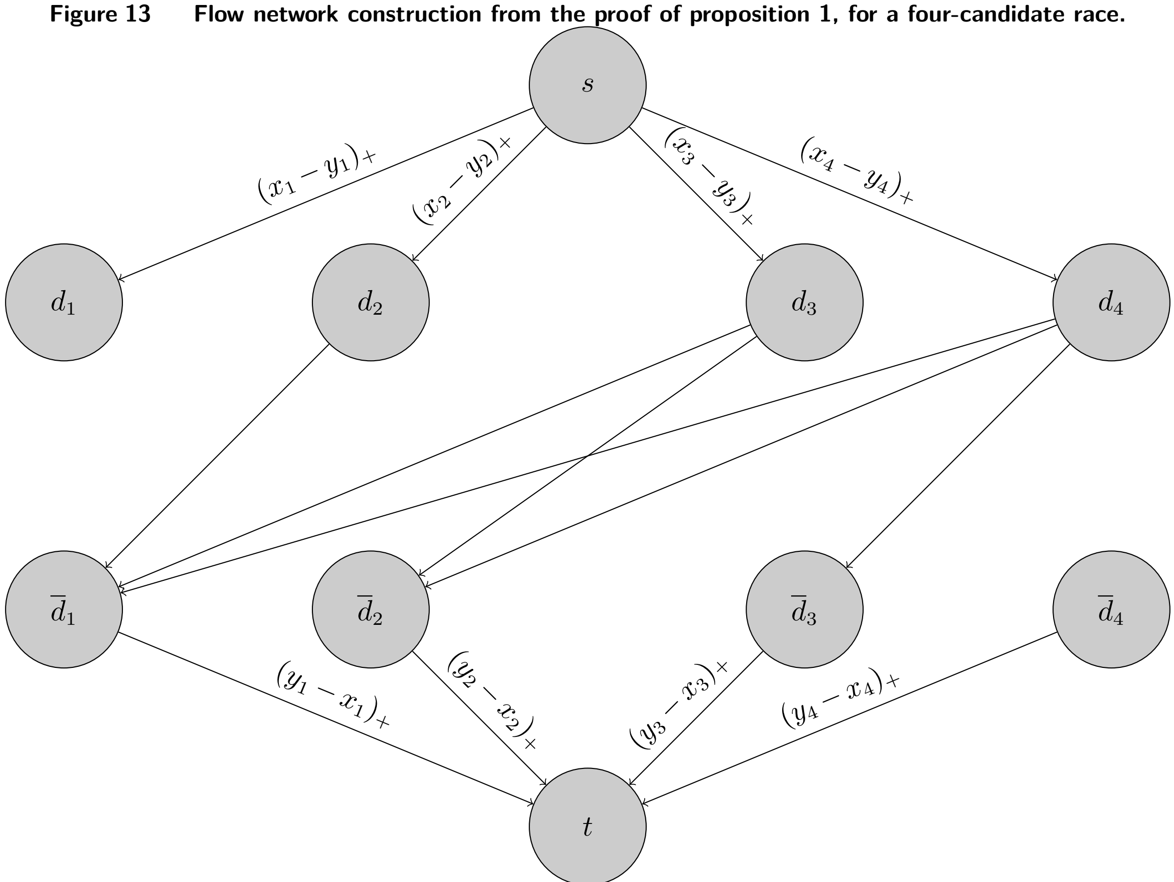
Belin, L. (2020). "Iowa Democrats should not certify inaccurate caucus results." *Bleeding Heartland* blog. Retrieved July 14, 2023, https://www.bleedingheartland.com/2020/02/28/iowa-democrats-should-not-certify-inaccurate-caucus-results/.

Berg, S. (1985). Paradox of voting under an urn model: the effect of homogeneity. *Public Choice 47*(2):377-387.

Berg, S. and Lepelley, D. (1992). Note sur le calcul de la probabilité des paradoxes du vote. *Mathématiques informatique et sciences humaines 120*: 33-48.

Berge, C. *Graphes et hypergraphes*, Paris, Dunod, 1970. oclc:301677997

Figure 13 **Flow network construction from the proof of proposition 1, for a four-candidate race.**



There is an up-preference transfer of delegates from x to y exactly when y is preferable to x. The flow determines the number of delegates transferred. By construction, only up-preference flow is permitted. For instance, if $y=(2,2,0,2)$ and $x=(1,2,1,2)$, then for $j=1,\ldots,4$, the inequalities

$$\sum_{i=1}^{j} y_i \geq \sum_{i=1}^{j} x_i$$

are all satisfied, so the cut $\big(\{s\}, G\setminus\{s\}\big)$ separating s from the first layer is a minimum cut, and y is a preferable delegate assignment with respect to the canonical candidate preference. One delegate can be transferred from the third candidate to the first in moving from x to y; the corresponding flow in the graph follows the path $s\to d_3\to \overline{d}_1\to t$ and has value 1, which is the maximum flow in the network.

Biesecker, M., and Slodysko, B. (2020). "Maker of glitchy Iowa caucus app has Democratic Party ties." Retrieved July 14, 2020, *The Associated Press*. https://apnews.com/article/technology-iowa-election-2020-campaign-2016-5232ce5601996c1de440806ad30fa4fb

Coll, Joseph A. (2021) "Demographic disparities using ranked-choice voting? Ranking difficulty, under-voting, and the 2020 Democratic primary." *Politics and Governance, 9*(2), 293-305.

Day, M. (2020). "Bernie Won Iowa." *Jacobin*. Retrieved July 14, 2023, https://jacobin.com/2020/02/bernie-sanders-iowa-caucus-democratic-presidential-primary

Democrats Abroad (2023). "Democrats Abroad Delegate Selection Plan for the 2024 Democratic National Convention." Retrieved July 14, 2023, https://drive.google.com/file/d/1JiC-

H2ngZQOSwZEdsFuByKrX-gRcN9Rp/view.

DNC (2022). “Delegate Selection Rules for the 2024 Democratic National Convention.” Accessed July 14, 2023, https://democrats.org/wp-content/uploads/2023/03/2024-Delegate-Selection-Rules.pdf

De Wolff, D., Kremnitzer, B., Perlman, S., & Weick, G. (2021). Selecting Representative and Qualified Candidates for President: Proposals to Reform Presidential Primaries. Fordham University, the School of Law.

Dickinson, T. (2020). “Popular Vote Proponent Pete Buttigieg Claims Victory in State Where Another Guy Got More Votes.” *Rolling Stone*. Retrieved July 14, 2023, https://www.rollingstone.com/politics/politics-news/iowa-caucus-pete-buttigieg-claims-victory-bernie-sanders-949544/

FairVote (2023). “Research and data on RCV in practice.” Retrieved September 4, 2023, https://fairvote.org/resources/data-on-rcv/.

Fedor, L. (2020). “Iowa party declares Pete Buttigieg winner of Democratic caucuses.” *Financial Times*. Retrieved July 14, 2023, https://www.ft.com/content/c220f204-490a-11ea-aeb3-955839e06441

Gallagher, M. (1992). Comparing proportional representation electoral systems: Quotas, thresholds, paradoxes and majorities. *British Journal of Political Science, 22*(4), 469-496. doi:10.1017/S0007123400006499

Patrick Horst. (2021). Die Präsidentschaftsvorwahlen der Demokraten 2020: Eine Rückkehr zur parteielitengesteuerten Kandidatennominierung vor 2016? *ZParl Zeitschrift für Parlamentsfragen, 52*(2), 245-263.

The Iowa Legislature (2016). “Map of the Week: Voter Turn Out—2014 General Election.” Retrieved July 14, 2023, https://www.legis.iowa.gov/docs/publications/MOW/attachments/767061_941718.jpg

Jamieson, Amber (2020). “Warren And Klobuchar Immediately Went After Bloomberg For His Past Sexist Comments At The Debate,” *Buzzfeed News*. Retrieved July 30, 2023, https://www.buzzfeednews.com/article/amberjamieson/warren-klobuchar-bloomberg-women-sexism

Caitlin E. Jewitt. (2019). Restoring Trust and Reducing Perceived Influence: Superdelegates and the 2020 Democratic Nomination. *Society 57*, 680-685.

Jones, M.A., McCune, D., & Wilson, J. (2019). “The elimination paradox: apportionment in the Democratic Party.” *Public Choice, 178*(1-2), 53-65. doi:10.1007/s11127-018-0608-3

Jones, M.A., McCune, D., and Wilson, J. (2023). *Delegate Apportionment in the US Presidential Primaries: A Mathematical Analysis*. Springer International Publishing, Cham, Switzerland. doi:10.1007/978-3-031-24954-9

Kamwa, Eric. (2019). “On the Likelihood of the Borda Effect: The Overall Probabilities for General Weighted Scoring Rules and Scoring Runoff Rules. *Group Decision and Negotiation, 28*(3):519-541. hal:hal-01786590f.

Koebler, J., Cox, J., and Maiberg, E. (2020). "An 'Off-the-Shelf, Skeleton Project': Experts Analyze the App That Broke Iowa." *Motherboard.* Retrieved July 14, 2023, https://www.vice.com/en/article/3a8ajj/an-off-the-shelf-skeleton-project-experts-analyze-the-app-that-broke-iowa

Lepelley, D., and Valognes, F. (2003). Voting rules, manipulability and social homogeneity. *Public Choice, 116*(1-2): 165-184.

Linkletter, D (2020). "The Iowa Caucuses Were a Complete Mess—and We Did the Math to Prove It." *Popular Mechanics.* Retrieved July 14, 2023, https://www.popularmechanics.com/science/math/a30810883/iowa-caucuses-math-errors/.

Liu, B., Mahallati, N., & Turner, C. (2021). Ranked-Choice Voting Delivers Representation and Consensus in Presidential Primaries. *SSRN*:3822879.

Morning Consult (2020). "Sanders, Biden Can Expect Near-Equal Gain From Warren's Exit." Accessed July 30, 2023, https://pro.morningconsult.com/articles/sanders-biden-can-expect-near-equal-gain-from-warrens-exit

The Nation editorial board (2020). "'The Nation' Endorses Bernie Sanders and His Movement." Retrieved July 14, 2020, https://www.thenation.com/article/politics/endorsement-2020-bernie-sanders/.

North Dakota Democratic Party (2023). "North Dakota Delegate Selection Plan for the 2024 Democratic National Convention." Retrieved July 14, 2023, https://demnpl.com/wp-content/uploads/2023/03/2024-North-Dakota-Delegate-Selection-Plan.pdf.

Parscale, B. (2020). Tweet: `Quality control = rigged?` 🤔. Quote tweet of a video tweet from user `@FrancisBrennan` with the following text:

`MSNBC's Steve Kornacki reports:`

`"We've heard from the Iowa state Democratic Party... They say that they are doing, quote, 'quality control on the results that they've received.' They say they are doing it, quote, 'out of an abundance of caution." #RiggedElection?`

Retrieved July 14, 2023, https://twitter.com/parscale/status/1224533010890002434.

Twitter account `@RealKHiveQueenB` (2020). Tweet:

`When Hillary got 4,000,000 more votes, he cried "RIGGED!"`

`Such a fucking hypocrite.`

`` `Because I got 6,000 more votes': Bernie Sanders declares victory in Iowa caucus https://rawstory.com/2020/02/because-i-got-6000-more-votes-bernie-sanders-declares-victory-in-iowa-caucus/ #politics #feedly ``

Retrieved July 14, 2023, https://twitter.com/RealKHiveQueenB/status/1225517394241695747.

Twitter account `@Real_Talk_News1` (2020). Tweet. Retrieved July 14, 2023, https://twitter.com/Real_Talk_News1/status/1224926133239582723

Richie, R., Oestericher, B., Otis, D., & Seitz-Brown, J. (2021). Lessons from the use of ranked choice voting in American presidential primaries. *Politics and Governance, 9*(2), 354-364.

Roberts, D. (2016). “Bernie Sanders wants raw vote count released after tight finish in Iowa caucuses.” *The Guardian*. Retrieved July 14, 2023, https://www.theguardian.com/us-news/2016/feb/02/bernie-sanders-requests-vote-count-tight-finish-iowa-caucus-clinton

Rolland, A., Aubin, J. B., Gannaz, I., and Leoni, S. (2022). A Note on Data Simulations for Voting by Evaluation. hal:hal-03194218.

Sánchez, J. (2020). Revisiting McGovern-Fraser: Party Nationalization and the Rhetoric of Reform. *Journal of Policy History, 32*(1), 1-24. doi:10.1017/S0898030619000253

Trump, D.J. (2020). Tweet: `The fix is in... AGAIN. And we get to watch it play out on live TV. Incredible. #IowaCaucuses`. Retrieved July 14, 2023, https://twitter.com/DonaldJTrumpJr/status/1224552283242934272.

Trump, E. (2020). Tweet: `Mark my words, they are rigging this thing... what a mess. This is why people don’t want the #Dems running our county. #meltdown`. Retrieved July 14, 2023, https://twitter.com/EricTrump/status/1224553451247546368.

Hugh Winebrenner. (1983). The Evolution of the Iowa Precinct Caucuses, *The Annals of Iowa 46*(8), 618-635. doi:10.17077/0003-4827.8941.

Withnall, A. (2020). “Super Tuesday: Warren called on to drop out and back Sanders after bruising defeat.” Retrieved July 14, 2023, https://www.independent.co.uk/news/world/americas/us-election/super-tuesday-warren-drop-out-bernie-sanders-delegates-states-a9373821.html.